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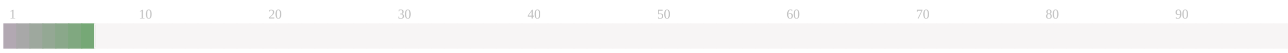

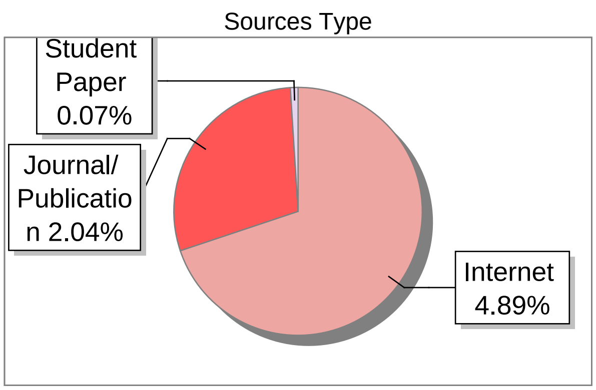

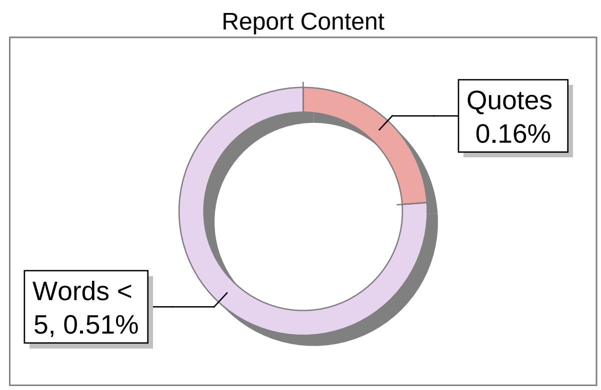

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QUANTITATIVE METHODS

[MBA]

(Credits: Theory-04)

Module 1

Basic numerical aptitude - Averages; Percentage and Ratio & Proportion

Module 2

Equations and line segments - Linear equations, Quadratic equations, Equation of Straight Line, Line Segment, Section Ratio and Gradient of a Line.

Commercial Mathematics- Profit & Loss, Computation of Interest, Annuities, Mensuration.

Module 3

Introduction to Data Analysis- Organizing, grouping, classifying, tabulation and graphical representation of numerical data, Introduction to Measures of Central Tendency and Measures of Dispersion.

Module4

Advanced Mathematical Applications - Set Theory and its application, Permutation and Combination, Matrix Algebra.

Module5.

Concept of probability, Meaning, definition, and applications of probability, introduction to linear programming- simplex method, Inventory models- determining EOQ and EOQ with price breaks.

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1.6 Percentages in Daily Life and Business

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1.1 Introduction

The concept of *average* is fundamental in statistics and mathematics. It represents a measure of central tendency, which aims to describe the center of a data set. There are different ways to calculate an average depending on the type of data and the context.

The concept of percentage is fundamental in mathematics and daily life, offering a way to express quantities as parts of a whole, represented out of 100. Derived from the Latin word "per centum," meaning "by the hundred," a percentage is a versatile tool used to compare, analyze, and calculate proportions. It simplifies complex relationships, making it easier to understand variations in data, such as changes in prices, grades, population growth, or profit margins. By expressing numbers as percentages, we can quickly interpret relative values and changes, making it an essential concept in fields ranging from finance and statistics to education and everyday decision-making. Understanding percentages is not only crucial for academic purpose but for scholarly use and real world applications, enabling individuals to navigate real-world scenarios with confidence.

Unit 1.2 Unit objectives

In this explanation, we will cover:

- The Concept of Average
- Basic questions on average
- Types of Averages: Mean, Median, and Mode
- Concept based on true or false average
- Concept of percentage
- Application on percentage

Unit 1.3 Introduction to Average Concept and calculations

1. The Concept of Average

The average is a collection of numbers in a single value that is utilized to represent the center or "typical" value in the data set. In everyday life, we often use averages to summarize a group of numbers or to compare data points.

Mathematically, an average is a method of calculating the central value of a dataset. The most common type of average is the **arithmetic mean**.

1.4 Different types of average, finding the mean, media and mode

1. Types of Averages

three main types of averages used in statistics are:

a. Mean (Arithmetic Mean)

The **mean** is the most common average used to represent a data set. It is calculated by summing all the values in the dataset and dividing by the number of values.

Formula:

Mean (μ or x) = n $\sum xi$

Where:

$\sum xi$ = Sum of all values in the data set.

n = Total number of values.

b. **Median** : the median is the middle value of a dataset when the values are arranged in ascending or descending order. when a value appears an uneven number of times, the median is the middle one; if there is any even quantity exists, the median is the average of the two middle values.

Steps to find the Median:

1. Arrange the data in ascending or descending order.
2. If the number of values (n) is odd, the median is the middle value.
3. If the number of values (n) is even, the median is the average of the two middle values.

c. Mode

The mode is the value occurring most often in a dataset. A dataset may have:

- One mode (unimodal),
- More than one mode (bimodal or multimodal),
- Or no mode if all values are unique.

Solved examples:

Find the mean of the following dataset.

5, 8,12,15,20

Solution:
To calculate the mean:

1. Add all the values: $5 + 8 + 12 + 15 + 20 = 605\ +\ 8\ +\ 12\ +\ 15\ +\ 20$
$= 605 + 8 + 12 + 15 + 20 = 60$
2. Divide by the number of values $(\mathrm{n}\ =\ 5)$:

$$\text{Mean} = 60/5 = 12$$

Answer: The mean is 12.

Example 2: Median Calculation

Problem:
Find the median of the following dataset:
7,3,9,2,57, 3, 9, 2, 57,3,9,2,5

Solution:

1. Arrange the data in ascending order: 2,3,5,7,92, 3, 5, 7, 92,3,5,7,9

2. The number of values (n = 5) is odd, so the median is the middle value: Median=5
3. **Answer:** The median is 5.

Example 3: Mode Calculation

Problem:
Find the mode of the following dataset:
4,6,8,4,9,6,44, 6, 8, 4, 9, 6, 44,6,8,4,9,6,4

Solution:

1. Count the frequency of each number:

- 4 appear 3 times.
- 6 appear 2 times.
- 8 appear 1 time.
- 9 appear 1 time.

2. The value with the highest frequency is 4, which appears 3 times.

Answer: The mode is 4.

Example 4: Median with Even Number of Values

Problem:
Find the median of the following dataset:
11,7,5,2,9,811, 7, 5, 2, 9, 811,7,5,2,9,8

Solution:

1. Arrange the data in ascending order: 2,5,7,8,9,112, 5, 7, 8, 9, 112,5,7,8,9,11
2. The number of values (n = 6) is even, so the median is the average of the two middle values (7 and 8): Median=7+8/2=15/2=7.5

   Median=27+8=215=7.5

Answer: The median is 7.5.

1. Mean (Arithmetic Mean) Examples

Example 1: Finding the Mean of a Small Dataset

Problem:
Find the mean of the following dataset:
10,12,15,20,2510, 12, 15, 20, 2510,12,15,20,25

Solution:
To calculate the mean:

1. Add all the values: 10+12+15+20+25=8210 + 12 + 15 + 20 + 25 = 8210+12+15+20+25=82
2. Divide by the number of values (n = 5):

   Mean=82/5=16.4

Answer: The mean is **16.4**.

Find the mean of the following dataset:
5,7,12,14,18,22,25,30,355, 7, 12, 14, 18, 22, 25, 30, 355,7,12,14,18,22,25,30,35

Solution:

1. Add all the values: 5+7+12+14+18+22+25+30+35=1685 + 7 + 12 + 14 + 18 + 22 + 25 + 30 + 35 = 1685+7+12+14+18+22+25+30+35=168
2. Divide by the number of values (n = 9):

   Mean=168/9=18.67

Answer: The mean is **18.67**.

Mean with Negative Numbers

Problem:
Find the mean of the following dataset:
−5,2,8,−3,6-5, 2, 8, -3, 6−5,2,8,−3,6

Solution:

1. Add all the values: −5+2+8+(−3)+6=8-5 + 2 + 8 + (-3) + 6 = 8−5+2+8+(−3)+6=8
2. Divide by the number of values (n = 5):
3. Mean=8/5=1.6

Answer: The mean is **1.6**.

Median Examples

Example 1: Finding the Median (Odd Number of Values)

Problem:
Find the median of the following dataset:
11,6,3,19,211, 6, 3, 19, 211,6,3,19,2

Solution:

1. Arrange the data in ascending order: 2,3,6,11,192, 3, 6, 11, 192,3,6,11,19
2. The number of values (n = 5) is odd, so the median is the middle value: Median=6

Answer: The median is **6**.

Formula for Mode (Concept)

There is no fixed formula for calculating the mode, as it is determined based on the frequency of the values in a dataset. However, you can follow these steps to calculate the mode:

1. **Sort the Data**: Arrange the data in ascending or descending order (though this step is optional).
2. **Count the Frequency**: Count the frequency of each value in thedataset.
3. **Identify the Most Frequent Value(s)**:

   - **In the case that there is one value with the highest frequency**, that value is the **mode**.
   - **If the multiple values possess the same highest frequency**, the dataset is **multimodal**, and all those values are modes.
   - When all values occur with equal frequency, a mode does not exist.

Finding the Mode (Bimodal)

Problem:
Find the mode of the following dataset:
4,5,7,7,8,10,10,10,124, 5, 7, 7, 8, 10, 10, 10, 124,5,7,7,8,10,10,10,12

Solution:

1. Count the frequency of each value:

   - 4 appear 1 time.
   - 5 appear 1 time.
   - 7 appear 2 times.
   - 8 appear 1 time.
   - 10 appear 3 times.
   - 12 appear 1 time.

2. The values 10 and 7 both appear most frequently (3 times and 2 times, respectively), so the dataset is bimodal.

Answer: The modes are **7** and **10**.

Summary of Key Points

- Mean: The average value, calculated by summing all values and dividing by the number of values.
- Median: The middle value in a sorted dataset. When the number of values is an even number of values, the result is the average of the two middle numbers.
- Mode: The most frequent value(s) in the dataset. A dataset can have one mode (unimodal), more than one mode (bimodal or multimodal), or no mode at all.

Test yourself.

Q1: Find the mean of 5, 10,15,20,25.

Q.2: Find the mean of the given data set: 10, 20, 30, 40, 50,60,70,80,90.

Q3. Find the mean of the first 10 even numbers.

Q.4: Find the mean of the first 10 odd numbers.

Basic questions on average:

Q1. The average age of A, B and C is 26 years, if the average age of A and C is 28 years, what is the age of B in years.

Q2. The average age of 7 numbers is 5. If the average age of first six of these numbers is 4, the seventh number is ?

Q3. The average age of 10 numbers is 7. What will be the new average if each of these numbers is multiplied by 8?

Q4. The average of five consecutive even numbers starting with 4 is?

1.5 : Concept to Calculate the Correct Average after Incorrect Data Entry

- **Identify the Error**:
- Compare the wrong data with the correct data (if available) to pinpoint the error(s).
- For example, a student's score may have been entered as 90 instead of 80.
- **Calculate the Incorrect Average** (if the incorrect data is already given):
  Find the average using the incorrect marks.
  Use the formula for **mean (average)**:

$$\text{Incorrect Mean} = \frac{\text{Sum of Incorrect Marks}}{\text{Total Number of Students}}$$

1. **Correct the Data**:

- Replace the incorrect marks with the correct ones.
- Ensure that you have the correct data (e.g., replacing 90 with 80).

2. **Recalculate the Correct Average**:

- After correcting the errors in the data, calculate the average using the correct values.
- Use the same formula for the mean:
- Correct Mean=$\frac{Sum\ of\ Correct\ Marks}{Total\ Number\ of\ Students}$

3. Compare the Old and New Averages:

Q. The difference between the incorrect and correct average will give you an idea of how the errors affected the overall result.

Example

Q1. The average of 100 observations is 40. Two items were incorrectly counted as 30 and 27 at the time of computation, rather than 3 and 72. Determine the accurate mean.

Solution: Total/100 = 40

total = 4000

incorrect numbers =30+27 = 57,

correct numbers = 3+72 = 75
corrected total = total - incorrect number + correct number
= 4000 - 57 + 75

= 4018

Hence

The correct mean = total / number

= 4018/100

= 40.18

Q2. It was discovered that the average of 100 items was 30. Determine the correct mean if two observations were incorrectly interpreted as 32 and 12 rather than 23 and 11.

Solution: Formula used

Mean = Sum of total observation(x) /Total Observation (n)

Given Mean = 30

Number of observation = 100

Sum of total observation = 3000

Incorrect value of x = 3000
Correct value of x = 3000 - (25 + 13) + (24 + 12)
= 3000 - 38 + 36
= 2998

Correct Mean = Correct value of number of Observation

= 2998100
= 29.98

Q3. Arithmetic mean of 9 observations was calculated as 45. In doing so, an observation was mistakenly taken as 42 instead of 24. What would then be the correct mean?

Solution:

The mean of 9 observations is 45.

Thus, the sum of the 9 observations is 45 × 9 = 405.

However, during the calculation of the mean, 42 was used instead of 24.

Therefore, the corrected sum of the 9 observations is 405 - 42 + 24 = 387.

Therefore, Actual mean of 9 observations $=\sum\frac{Xi}{N}$

$=\frac{387}{9}=43$

Q4. The average of a set of 75 observations was initially determined to be 27. Later, it was discovered that one value had been mistakenly recorded as 43 instead of the correct value of 53. Calculate the accurate average of the data.

Solution:

Number of observations= 75

Mean=27

Sum of observations=75×27

Wrong value=43, Correct value=53

∴Correct sum=75×27−43+53=75×27+10

Correct mean=$75\times27+1075=27+1075=27.133$

Q5. The arithmetic mean of 10 numbers was computed as 7.6. It was subsequently discovered that a number 8 was wrongly read as 3 during the computation. What should be the correct mean?

Solution: Arithmetic mean of 10 numbers is 7.6.

Therefore, sum of numbers $=10\times7.6=76$

8 were wrongly read as 3 during computation.

Therefore, correct sum of numbers $=76+8-3=81$

Correct mean $=81\div10=8.1$

Test yourself:

Q1. The arithmetic mean of 10 numbers was computed as 7. It was then found that a number 9 was wrongly read as 3 during the computation. What should be the correct mean?

Q2. Nine observations' arithmetic mean was determined to be 54. As a result, an observation was incorrectly interpreted as 32 rather than 23. Then, what would the proper mean be?

Q3. The mean of 4 numbers was computed as 7, but Later, it was on found that 3 was misread as 8, the actual mean of this data is ?

Q4. The mean of 4 numbers was computed as 7, but Later, it was on found that 3 was misread as *8,* the actual mean of this data is?

Q5. The mean of the marks obtained by 100 students is 60. If the marks obtained by one of the students were incorrectly calculated as 75, whereas the actual marks obtained by him were 65, what is the correct mean of the marks obtained by the students?

Q6. A mathematics teacher tabulated the marks secured by 35 students of 8h class. The average of their marks was 72. If the marks secured by Reema was written as 36 instead of 86 then find the correct average marks upto two decimal places.

Q8. Average marks of 14 students was calculated as 71.but, later it was found that one of marks was wrongly entered as 42 instead of 56 and of another as 74 instead of 32. The correct average is.

Q.9. The average marks in science subject of a class of 20 students is 68. If the marks of two students were misread as 48 and 65 of the actual marks 72 and 61 respectively, then what would be the correct average?

Q.10. The average marks of English subject of a class of 24 students is 56. If the marks of three students was misread as 44,45 and 61 of the actual marks 48,59,67 respectively , then what would be the correct average.

Concepts on average speed

Average speed is a measure of how quickly something moves over a given distance during a specified period of time. It is calculated by dividing the total distance traveled by the total time taken. Average speed is a scalar quantity, meaning it only has magnitude and no direction.

The formula can be written as:

Average speed = $\frac{total\ distance}{total\ time}$

Example :

If a car travels a distance of 200 km in 4 hrs. the average speed is

Average speed = $\frac{total\ distance}{total\ time}$ = $\frac{200}{4}$ = 50 km/hr

This means that the car is travelling with a average speed of 50 km/hr.

Questions based on average speed.

Q.1 a man goes to a certain place at a speed of 30 km/hr and returns to original place at a speed of 20km/hr . calculate the average speed during the whole journey.

Q2. A train covers the first 160 km at a speed of 120 km/hr, another 160 km at 140 km/hr and last 160 km at 80 km/hr. find out the average speed of the train for the entire journry.

Q.3 a person covers 9km at a speed of 3km/hr, 25km at a speed of 5km/hr, and 30km at a speed of 10km/hr. find out the average speed for the entire journey.

Questions based on equation:

Q1. The average of marks scored by 120 candidates was 35. If the average of marks of the passed candidates was 39 and that of failed candidates was 15,the number of candidates who passed the examination is:

Q2. In a school, average age of students is 6years and the average age of 12 teacher is 40 years . if the average age of the entire student and teacher group is seven years. Then the no. of students is?

Q3. The average monthly salary of all the employees in an industry is Rs.12000. the average salary of male employees is Rs.15000 and that of female employee is Rs. 8000. what is the ratio of male employees to the female employees?

Q4. In a school of 600 students, the average age of boys is 12 years and that of girl is 11 yearsHow many girls are there in the school if the average age of the students is eleven years and nine months?

Q.5 the average salary of all the staff in a office of a corporate house is Rs.5000. the average salary of the officers is Rs.14000 and among the remaining is Rs.4000. If there are 500 employees overall, how many officers are there?

Unit 1.6 Understanding of percentage:

Views of Percentage and Its Conversion to Fractions

Percentage is a method of representing a number as a part of 100. The term "percent" is derived from the Latin expression per centum, which translates to "out of one hundred."

Definition:

A **percentage** is a ratio expressed as a fraction of 100.

For example, **50%** means **50 out of 100**, or simply **50/100**.

Basic Formula for Percentage

$$\text{Percentage} = \frac{\textit{The quantity to be expressed in percentage}}{\textit{2nd quantity(in respect of which the percent has to be obtained}} \times 100\%$$

This formula helps to calculate the percentage when the part and the whole are known.

Conversion of Percentage to Fraction

To convert a percentage to a fraction, follow these steps:

1. Write the percentage number as the numerator.
2. Use 100 as the denominator.
3. Simplify the fraction if possible.

Let us look some of the percentage amounts, and their fractions and decimal equivalents

75%$= \frac{75}{100} = \frac{3}{4} =$ **0.75**

50%$= \frac{50}{100} = \frac{1}{2} =$ **0.50**

25%$= \frac{25}{100} = \frac{1}{4} =$ **0.25**

10%$= \frac{10}{100} =$ **0.1**

5% $= \frac{5}{100} =$ **0.05**

Important fraction to percentage conversion

$\frac{1}{2}$	**50%**
$\frac{1}{3}$	**33.33%**
$\frac{1}{4}$	**25%**
$\frac{1}{5}$	**20%**
$\frac{1}{6}$	**16.66%**
$\frac{1}{7}$	**14.28%**
$\frac{1}{8}$	**12.5%**
$\frac{1}{9}$	**11.11%**
$\frac{1}{10}$	**10%**
$\frac{1}{11}$	**9.09%**

$\frac{1}{12}$	**8.33%**
$\frac{1}{13}$	**7.69%**
$\frac{1}{14}$	**7.14**
$\frac{1}{15}$	**6.66%**
$\frac{1}{16}$	**6.25%**
$\frac{1}{17}$	**5.88%**
$\frac{1}{18}$	**5.55%**
$\frac{1}{19}$	**5.26%**

How to convert fraction to percentage and vice versa

From the explanation above, we understand that to convert a fraction into a percent, we first convert it into an equivalent fraction with a denominator of 100 and then add the percentage symbol (%) to the new numerator.

$\frac{3}{4} = \frac{3}{4} \times 100 = 75\%$

$\frac{4}{25} = \frac{4}{25} \times 100 = 16\%$

$\frac{4}{25} = \frac{4}{25} \times 100 = 16\%$

Note: To express a fraction as a percent, we can multiply the fraction by 100, simplify the result, and then add the % symbol. For example,

Conversely,To express a percent as a fraction, we drop the % sign, multiply by $\frac{1}{100}$(or divide the number by 100) and simplify it, for example

$47\% = 47 \times \frac{1}{100} = \frac{47}{100}$, $17\% = 17 \times \frac{1}{100} = \frac{17}{100}$, $45\% = 45 \times \frac{1}{100} = \frac{45}{100} = \frac{45}{100} = \frac{9}{20}$

Method for conversion of decimal into a percent and vice versa

Let us consider the following examples;

$0.35 = \frac{35}{100} = 35 \times \frac{1}{100} = 35\%$

$4.7 = \frac{47}{10} = \frac{470}{100} = 470 \times \frac{1}{100} = 470\%$

$0.459 = \frac{459}{1000} = \frac{459}{10} \times \frac{1}{100} = 45.9\%$

Therefore, to convert a decimal into a percent, we shift the decimal point two positions to the right and add the % sign.

Conversely,

To convert a percent into a decimal, we remove the % symbol and shift the decimal point two places to the left. For instance,

43% = 0.43 75% = 0.75 12% = 0.12 9% = 0.09

0.75% = 0.0075 4.5% = 0.045 0.2% = 0.002

Let us try few more examples,

Example 1. Ram scored 18 marks in a test of 25 marks. Calculate his percentage of marks?

Solution : Total marks = 25

Marks scored = 18

Therefore, The portion of marks achieved.

$= \frac{18}{25}$

Marks scored in percent $= \frac{18}{25} \times \frac{4}{4} = \frac{72}{100} = 72\%$

1.7 Scope of Percentage and its applications:

Example1- A quarter of the total shoes in the store were on sale at a discount. What percentage of the shoes were sold at the regular price?

Solution: percentage of the total quantity of shoes on sale $= \frac{1}{4}$

Therefore, The proportion of the overall shoes sold at the regular price.

$= 1 - \frac{1}{4} = \frac{3}{4}$

$$= \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 75\%$$

Example2. In a class of 40 students, 32 chose to go on a picnic. What percentage of students decided to go on the picnic?

Solution : The overall number of students in a class is 40

The no. of students who chose to go on the picnic.

$= 32$

Therefore,

$= \frac{32}{40} \times 100\% = 80\%$

Example3. A mixture of 80 liters consisting of acid and water contains 20 liters of acid. What is the percentage of water in the mixture?

Solution: Solution: The overall volume of the mixture.

$= 80$litres

Volume of acid = 60 litres

Therefore, The proportion of water in the mixture.

$= \frac{60}{80} \times 100\% = 75\%$

Let's consider a few examples from everyday life.

Example: Amal scored 62% in an exam with a total of 600 marks. How many marks did Amal obtain?

Solution? Here, we have to find 62% of 600

Therefore, 62% of 600 $= 0.62 \times 600$ marks = 372 marks

Marks scored by Amal = 372

Example: Neha earns Rs. 30,800 each month. She allocates 50% for household expenses, 15% for personal expenses, and 20% for her children's expenses. What amount does she save monthly from the remaining money?

Solution. Expenses on household = 50%

Expenses on self = 15%

Expenses on children =20%

Total expenses =(50+15+20)% = 85%

Therefore , her savings (100 - 85)= 15%

Therefore , 15% of Rs 30800 = Rs. (0.15× 30800)

answer = Rs. 4620

1.8 Unit summary

1. Finding the Whole When a Percentage is Given:

Question: If 30% of a certain number equals 90, Find the number?
Solution: Let the number be x We know that: 30%×x=90,
$\frac{30}{100} \times x = 90;\ x = 90 \times \frac{100}{30} = 300$

Answer: The number is 300.

2. Raising or lowering a number by a percentage.

Question: A shirt costs Rs120 and it is on sale for 25% off. What is the sale price?

Solution: First, find the discount amount: 25% of 120=25/100×120=30
Subtract the discount from the original price: 120−30=90
Answer: The price after discount is Rs 90.

3. Comparing Percentages:

Question: Which is greater, 60% of 200 or 3/4 of 240?

Solution: First, calculate 60% of 200=60% of 200=60/100×200=120
Then calculate ¾ of 240 which is 180

Answer = 180

4. Converting Decimal to Percentage:

Q. Convert 0.75 to a percentage.

Solution: 0.75×100=75%

Answer: 0.75 is equal to 75%.

5. Converting Percentage to Decimal:

Question: Convert 85% to a decimal.

Solution: 85/100=0.85

Answer: 85% is equal to 0.85.

What percentage formula:

Formula 1. X is what percent of y

$$= \frac{x}{y} \times 100\%$$

What percent of x is y = $\frac{y}{x} \times 100$

Formula 2. X is what percent more or less than y

$$\frac{x-y}{y} \times 100$$

Q1. If x is 10% more than y, then by what percent is y less than x?

(a) $9\frac{1}{11}\%$

(b) $7\frac{1}{11}\%$

(c) $8\frac{1}{11}\%$

(d) $10\frac{1}{11}\%$

Q2. If A's height is 10% more than B's height, by how much percent less is B's height than that of A?

Answer

(a) 10% (b) $10\frac{1}{9}$% (c) 15% (d) $9\frac{1}{11}$

Q.3 B got 20% less than A. what percent marks did A got more than B?

Answer. (a) 20% (b) 25% (c) 10% (d) 12%

Q4. If x earns 25% more than y. what percent less does y earns than x?

Answer. (a) 16% (b) 10% (c) 20% (d) 25%

Q3. A pair of numbers are respectively $12\frac{1}{2}$ % and 25% more than a third number. The first number is what percentage of the second number?

Answer: (a) 50% (b) 60% (c) 75% (d) 90%

Q.4 Two numbers are smaller than a third number by 30% and 37% respectively. What is the percentage by which the second number is less than the first?

Answer : (a)10% (b)7% (c)4% (d) 3%

Question based on salary

Q1. Radha spends 40% of her salary on food, 20% on house rent, 10% on entertainment and 10% on conveyance. If her savings at the end of a month are Rs.1500, then her salary per month(inRs.)is

Q2. Kishan spends 30% of her salary on food and donates 3% on a charitable trust. He spends Rs. 2310 on these two items , then total salary for that month is?

Q3. Mr.X spends 30% of his salary on food 5% of his salary on children education. In January 2011, he spent Rs.17600 on these two items .his salary for that month is ?

Q4. Arup spends Rs.55475 on his birthday party ,Rs.28525 on buying home appliances and the remaining 25% of his total amount he had has cash with him. What was the total amount?

Percentage based on voters

Q1.two persons contested an election of parliament. The winning candidate secured 57% of the total votes polled and won by a majority of 42000. The overall number of votes cast is?

Q2. In an election, a candidate received 40% of the votes but is defeated by only other candidate by a majority of 298 votes. Find the total number of votes recorded.

Q3. In a election between two contenders one get 72% of the total votes. If the total votes are 8200 by how many votes did the winner win the election.

Q4. In an election between two contestants, the candidate receiving 60% of the votes polled is elected by a majority of 14,000 votes. The number of votes polled by the winning candidates is.

Q5. In an election, three candidates contested. The first candidate got 40% votes and the second got 36% votes. If the total number of votes casted were 36000, find the number of votes got by the third candidates is?

Q6. In a college election between two candidates, 10% of the votes are invalid. The winner gets 70% of the valid votes and defeats the looser by 1800 votes. How many vote were totally cast?

Q7. In an election with two candidates, 8% of voters did not vote. The winner, securing 48% of the total votes, defeated the other candidate by 1,100 votes. What was the total number of voters in the election?

Unit 1.9 Check your progress

1. **Convert 5/8 to a percentage.**
2. **What is 15% of 80?**
3. **A population of 5000 increases by 12%. What is the new population?**
4. **Convert 0.2 to a percentage.**
5. **If 50% of a number is 25, what is the number?**
6. **Find the percentage increase if a price changes from \$100 to \$125.**
7. **Convert 1.25 to a percentage.**

Check your progress

1. Convert each of the following into a percent:
   a) $\frac{12}{25}$ b) $\frac{9}{20}$ c) $\frac{5}{12}$ d) $\frac{6}{15}$ e) $\frac{125}{625}$ f) $\frac{3}{10}$
2. Write each of the following percent's as a fraction:
   (a) 53% (b) 47% (c) 85% (d) $16\frac{2}{3}$ (e) $11\frac{1}{9}$ (f) 47.35%
3. Write each of the following decimals as a fraction:
   (a) 0.97 (b)0.635 (c) 0.03 (d)2.07 (e)0.8 (f)1.75
4. Sulekha obtained 18 marks in a test of total 30 marks. What percent was his percentage of marks?

Unit 2: RATIOS AND PROPORTION

Structure:

Unit 2.1 Introduction to Ratios

A **ratio** is a way of comparing two or more quantities. It shows how much of one thing there is in relation to another. Ratios are written as:

or **a/b** or **a to b**.

For example, if there are 2 apples and 3 oranges, the ratio of apples to oranges is **2:3**.

Key Points:

- The numbers in a ratio are called **terms**.
- The first number is called the **first term**, and the second number is called the **second term**.

Unit 2.2: Unit Objectives:

By completion of this unit, students will be able to:

1. **Understand the Concept of Ratios**
    - Define a ratio and express it in various forms (e.g., fraction, colon, and word form).
    - Identify and compare different types of ratios in real-life contexts.
2. **Simplify Ratios**
    - Simplify ratios to their lowest terms.
    - Solve problems involving equivalent ratios.
3. **Apply the Concept of Proportion**

- Understand the meaning of proportion and its relationship with ratios.
- Identify and verify proportional relationships between quantities.

4. **Solve Problems Involving Proportions**

   - Use cross-multiplication to solve proportion problems.
   - Apply proportional reasoning to solve word problems in contexts such as scaling, map reading, and recipe adjustments.

2.3 Understanding Ratios types and Basic definitions

Types of Ratios

- Part to Part Ratio: Compares different parts of a whole.
- Example: In a class of 20 students, if 12 are boys and 8 are girls, the ratio of boys to girls is 12:8 or 3:2.

- Part-to-Whole Ratio: Compares a specific part of a whole to the entire amount.
  Example: If there are 3 red balls and 7 blue balls, the ratio of red balls to the total number of balls is 3:10.

Simplification of Ratios

Similar to fractions, ratios can also be reduced. by dividing both terms by their **greatest common divisor (GCD)**.

For example:

24:36 can be simplified by dividing both numbers by 12 (GCD), so **24:36 = 2:3**.

Proportions

A **proportion** is an equation that expresses the equality of two ratios. It states that two ratios are equivalent.

A proportion is written as:

a/b = c/d or **a : b = c : d**.

In this scenario, **a** is to **b** as **c** is to **d**.

Key Points:

- Cross Multiplication: In a proportion a/b = c/d, we can use the property of cross multiplication: a × d = b × c.
- Example: If 2/5 = 6/x, cross-multiply: 2 × x = 5 × 6, so 2x = 30. Solving for x, we get x = 15.

Applications of Ratios and Proportions

Ratios and proportions have a wide range of applications. Some examples include:

a) Maps and scales drawings

Maps use ratios to represent distances in real life. For example, on a map, 1 cm might represent 100 km in reality. This is a **scale ratio**.

b) Time and distance

In speed calculations, the ratio of distance to time is the speed of an object.

- **Speed = Distance/Time**
- If a car travels 120 km in 3 hours, the ratio of distance to time is **120:3**, and the speed is **40 km/h**.

c) Mixtures and Allegations

In mixing problems (like mixing ingredients or solutions), the ratio is used to determine the fraction of different ingredients in the mixture.

- **Example**: If two solutions with concentrations of 10% and 20% are mixed to form a 15% solution, the proportion can be solved using the concept of alligation.

d. Sharing of Quantities

When dividing something in a particular ratio, the ratio helps to find how much of each part each person gets.

- **Example**: If Rs.400 is to be shared between two individuals in a 3:5 ratio, how would the amount be divided?

- the shares would be divided as:
    - Total parts = 3 + 5 = 8 parts
    - Value of 1 part = 400 / 8 = 50
    - First person gets 3 × 50 = $150
    - Second person gets 5 × 50 = $250

Q1: Simplification of Ratios

=> **18:24** can be simplified by dividing both terms by their GCD (6):

- **18 ÷ 6 = 3**
- **24 ÷ 6 = 4**
- The simplified ratio is **3:4**.

Q2: Proportion Solving

- **5/8 = x/16**.
- Cross multiply: **5 × 16 = 8 × x**, so **80 = 8x**.
- Solving for **x**: **x = 80 ÷ 8 = 10**.

Q3: Applications in Speed

- Speed = Distance / Time = **250 km / 5 hours = 50 km/h**.

Q4: Sharing in a Ratio

- Total parts = **2 + 3 = 5**.
- Value of 1 part = **1200 / 5 = 240**.
- First child receives **2 × 240 = $480**.
- Second child receives **3 × 240 = $720**.

Q5: Mixing Solutions

- Total parts = **3 + 5 = 8**.
- Value of 1 part = **400 / 8 = 50 liters**.
- First solution = **3 × 50 = 150 liters**.

Second solution = **5 × 50 = 250 liters**.

Q6: Proportions in Geometry

- The ratio of corresponding sides is **4:7**.
- Let the corresponding side in the larger triangle be **x**.
- Establish a proportion:4/7 = 12/x.
- Cross multiply: **4x = 7 × 12** so **4x = 84**.
- Solving for **x**: **x = 84 ÷ 4 = 21 cm**.

Example 1: Finding Speed from Distance and Time

Problem:
A car travels 150 kilometers in 3 hours. What is its speed?

Solution:
We understand that speed is the ratio of distance traveled to time taken, expressed as:

Speed=$\frac{distance}{time}$

Substitute the given values:

Speed=$\frac{150}{3}$ =50 km/h

So, the speed of the car is **50 km/h**.

Example 2: Finding Distance from Speed and Time

Problem:
If a bike travels at a speed of 20 km/h for 4 hours, what is the distance traveled?

Solution:
We apply the formula for distance:

Distance=Speed×Time

Substitute the given values:

Distance=20 km/h×4 hours=80 km

So, the bike travels **80 kilometers**.

Example 3: Finding Time from Distance and Speed

Problem:
A train travels 180 kilometers at a speed of 60 km/h. How much time does it take to reach its destination?

Solution:
We use the formula for time:

Time=$\frac{distance}{speed}$

Substitute the given values:

Time=$\frac{180\ km}{60km/h}$ =3 hours

So, the train takes **3 hours** to travel 180 kilometers.

2. Ratio and Proportion Problems in Time and Distance

Ratios are also helpful in comparing two or more situations where the speed, time, and distance are related. Here's how you can use **proportions** to solve such problems.

Example 4: Comparing Two Journeys

Problem:
Two cars travel the same distance of 240 kilometers. The first car travels at a speed of 60 km/h, and the second car travels at 80 km/h. How much time will the second car save compared to the first car?

Solution:
We can calculate the time taken by each car using the formula:

Time=$\frac{distance}{speed}$

For the first car:

$Time\ _1$=$\frac{240km}{60km/h}$

For the second car:

$Time\ _2$= $\frac{240\ km}{80\ km/hr}$= 3 hours

The second car saves:

Time saved=4 hours−3 hours=1

So, the second car saves **1 hour**.

Example 5: Solving Using Proportions

Problem:
If a cyclist travels 45 kilometers in 3 hours, how long will it take to travel 75 kilometers at the same speed?

Solution:
Here, we can set up a proportion since the speed (distance/time) is constant. Let the unknown time be x.

We know the relationship between distance and time for both journeys is proportional:

$\frac{45\ km}{3\ h} = \frac{75 km}{x\ h}$

Now, we solve for x:

45×x=75×3
45x = 225
X= $\frac{225}{45}$ = 5 hrs

So, it will take **5 hours** to travel 75 kilometers at the same speed.

Example 6: Inverse Proportions of Time and Speed

Problem:
A car travels 300 kilometers at a speed of 75 km/h. How long will it take if the speed is reduced to 60 km/h?

Solution:
In this case, **time** and **speed** are inversely proportional. If the speed decreases, the time taken will increase.

We can set up the proportion based on inverse proportionality:

$\frac{speed_1}{time_1} = \frac{speed_2}{time\ _2}$

Substitute the given values:

$\frac{75}{\frac{time}{1}} = \frac{60}{time_2}$

First, calculate the time taken when the speed is 75 km/h:

$time_1 = \frac{distance}{speed} = \frac{300}{75}$ = 4 hrs

Now, using the inverse proportion:

$$\frac{75}{4} = \frac{60}{time_2}$$

Cross-multiply to find Time2\text{Time}_2Time2:

75×Time2=60×4
$75 \times time_2 = 240$

$time_2 = \frac{240}{75} = 3.2$ hrs

Thus, it will take the car **3.2 hours** to travel the same distance at 60 km/h.

3. Time and Distance Ratio Summary

When solving time and distance problems using ratios:

- **Speed = Distance / Time**: If two of these quantities are given, you can calculate the third.
- **Proportions**: You can use proportions when comparing two situations, especially if the speed, time, or distance are directly related (as in constant speed) or inversely related (as in time and speed).
- **Inverse Proportions**: Time and speed are inversely proportional, meaning if speed increases, time decreases for the same distance, and vice versa.

Practice Questions

1. A runner covers 200 meters in 25 seconds. What is their speed in meters per second?
2. If a car travels 120 km in 2 hours, how far will it travel in 5 hours at the same speed?
3. A train travels 120 kilometers in 1.5 hours. How much time will it take to travel 240 kilometers?

Unit 2.4: Concepts of mixture and allegation

The **Mixture and Allegations** concept involves combining two or more different substances or solutions (often with different concentrations) to form a new mixture with a desired concentration. The ratio and proportion techniques help in determining the amounts of each substance or solution to mix.

There are two important aspects:

1. **Allegations (or Allegations Medial)**: A method used to find the average or mean concentration of a mixture.

2. **Allegations Alternate: A technique used to calculate the ratio in which two or more quantities (such as solutions or ingredients) need to be combined to obtain a specific outcome.**

3. Mean price

   The cost price of the final mixture is referred to as the average price.

   Allegation rule:-

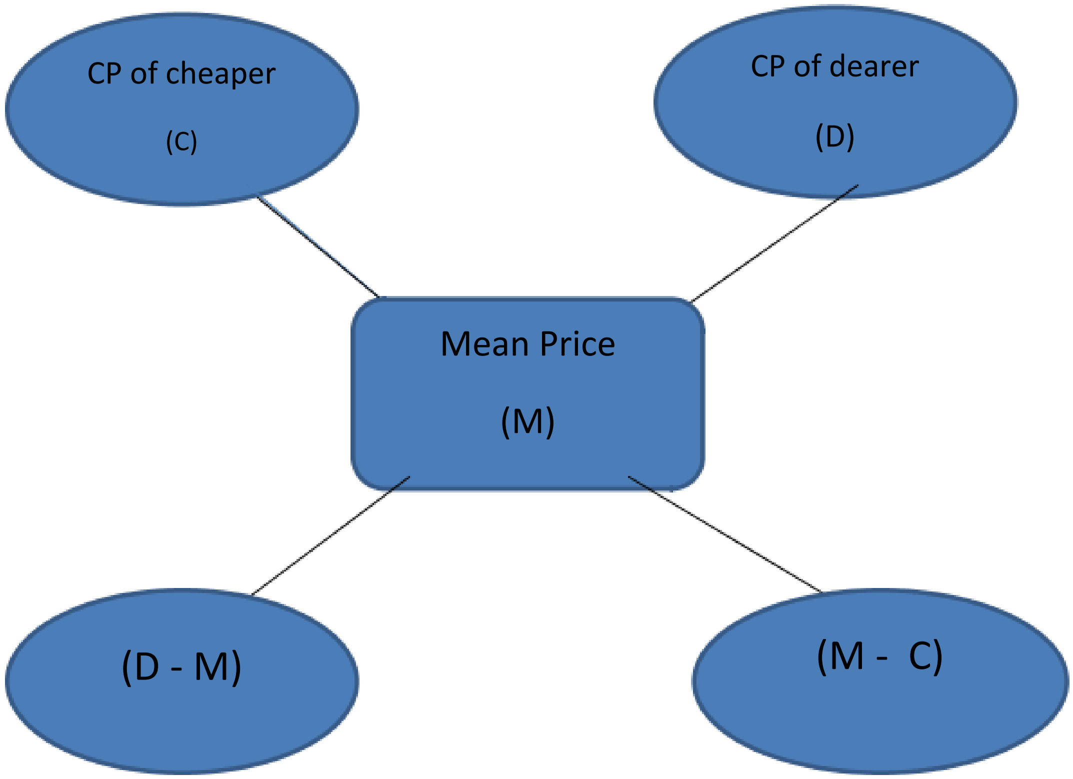
   When two elements are mixed to make and one of the elements is cheaper than the other one is costlier then

   we can use allegation by the formula:

   $$\frac{quabtity\ of\ cheaper}{quantity\ of\ dearer} = \frac{dearer - mean}{mean - cheaper}$$

   Here mean price is cost price of a mixture per unit quantity.

   Also rule can be written as



Points to remember when using the rule of allegation

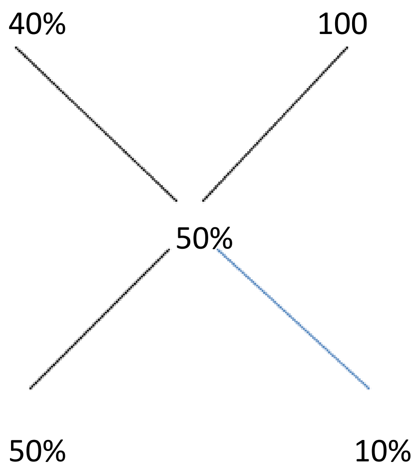
- The three values alighted should always represent the same variable and should have same units
- Allegation of three values of cost gives the ratio expressed as the number and vice versa.
- If two values of cost price and The selling price of the mixture is provided., then in such cases first calculate the cost price of the mixture and the alligate the three values of cost price.

A and B represent concentration if the numerical is based on mixing of solutions.

Ex1. 600 gm. of sugar solution has 40% sugar in it. How much sugar should be added to make it 50% of the solution?

Sol: The existing solution has 40% sugar, and sugar is to be mixed; so the other solution has 100% sugar. So, by alligation method?

SOLUTION:



The two mixtures must be combined in a ratio of 5:1.
Hence, the required amount of sugar is.

(600/5)* 1 = 120

Finding ratios

Q1. If A:B = 1/2:1/3 and B:C = 1/2 : 1/3, then A:B:C is

Q2. If a : b = 5:7 and c:d= 2a:3b, then ac:bd is

Q3. If a:5 = b:7 =c:8 then $\frac{a+b+c}{a}$ is equal to

Q4. If X/Y = (6/5), find the value $\frac{x2+y2}{x2-y2}$ is ?

Q5. If x:y= 8:9, then 5x – 4y = 3x + 2y

Q6. If x/2y = 6/7, the worth of $\frac{(x-y)}{(x+y)} + \frac{14}{19}$ is

Q7. If a : b = 2 : 3 and b:c= 4:5 is then a^2:b^2:bc is

Q8. If A : B = 2 : 3, B:C= 4:5 and C:D= 6:7,find the values of A:B:C:D?

Questions based on divided into parts

Q1. If Rs.1000 is divided between A and B in the ratio of 3:2, then A will receive?

Q2. If 78 is split into three parts that are in the proportion of 1:1 / 3:1 /6 the middle part is

Q3. Rs. 33,630 is distributed among A, B, and C in a way that the ratio of A's share to B's share is 3:7, and the ratio of B's share to C's share is 6:5. How much money does B receive?

Q4. Rs. 3,400 is distributed among A, B, C, and D in such a way that the ratios of the shares of A to B, B to C, and C to D are 2:3, 4:3, and 2:3, respectively. What is the total of B's and D's shares?

Q5. By mistake instead of dividing Rs.117 among A,B and the ratio ½: 1/3: ¼ it was divided in the ratio 2:3:4 . who gains the most and by how much

Questions based on numbers

Q1. The total of two numbers is 40 and difference between them is 4. Find the numbers.

Q2. Among three numbers, the ratio of the first to the second is 8:9, and the ratio of the second to the third is 3:4. If the product of the first and third numbers is 2400, what is the value of the second number?

Q3. Three numbers are in the ratio of $\frac{1}{2}:\frac{2}{3}:\frac{3}{4}$ The difference between the largest and smallest numbers is 36. What are the numbers?

Questions based on income and expenditure

Q1. The ratio of income of P and Q is 3:4 and their ratio of their expenditure is 2:3.if both of them save Rs.6000 each, then income of P is

Q2. A and B have monthly income in the ratio of 5:6 and monthly expenditures in the ratio 3:4 . if they save Rs. 1800 and 1600 repectively. What is B's monthly income?

Q3. A man divides his monthly income between expenses and savings, maintaining a ratio of 26:3. If his total monthly income is Rs. 7,250, how much does he save each month?

Q4. The monthly salary of A, B ,C is in the proportion of 2:3:5. If C’s monthly salary is 12000 more than that of A, then B’s annual salary is.

Q5. The incomes of A and B are in the ratio of 5:3. The expenses of A,B,C are in the ratio of 8:5:2. If C spends Rs.2000. and B saves Rs 700, then A saves.

Questions based on ages

Q1. The ratios of the ages of two students is 3:2 one is elder than the other is 5 years who is the younger student.

Q2. The ages of two boys are in the ratio 5:6. In two years, the ratio will change to 7:8. What will their age ratio be after 12 years?

Q3. Harsha is 40 years old and Ritu is 60 years old. How many years ago was the ratio of the ages 3:5

Q4. The ratio of present age of two brothers is 1:2 and 5 years back the ratio was 1:3. What will be the ratio of ages after 5 years?

2.6 unit summary

The unit on ratio and proportion explores the foundational concepts of comparing quantities and understanding their relationships. A ratio expresses the relative size of two quantities, enabling comparisons in various contexts, such as measurements, finance, and everyday problem-solving. Proportion builds on this by identifying and solving equations that demonstrate the equality of two ratios. Students learn to simplify ratios, recognize equivalent forms, and apply proportional reasoning in both direct and inverse relationships. Real-world applications, such as scaling, map interpretation, and recipe adjustments, are integrated to enhance practical understanding. By mastering ratio and proportion, students develop critical thinking and problem-solving skills, equipping them to tackle mathematical and real-life challenges effectively.

Unit 2.7: Check your progress:

Q) A jeweler combines silver and gold in a 3:2 ratio to craft a necklace. Gold costs Rs. 3000 per gram, and the cost of the mixture is Rs. 1950 per gram. What is the price of silver per gram?

Q) The acid concentrations in the three containers are as follows: 60% acid is present in the first, 50% in the second, and 40% in the third. How much acid will be in the final mixture if these containers are combined in a 3:4:3 ratio?

QTwo containers have water and milk mixed in the ratios 2:3 and 1:2, respectively. What should be the mixing ratio of the contents from these two containers to obtain a new mixture with a specific water-to-milk ratio?

UNIT 3: Linear equations

Structure:

3.1 Introduction

3.2 Unit objectives

3.3 Basics of Linear Equations: Forms, Solutions, and Graphical Representation

3.4 Applications of Linear Equations in Real-World Scenarios

3.5 Introduction to Quadratic Equations: Forms, Solutions

3.6 Solving and Applying Quadratic Equations in Practical Situations

3.7 Unit summary

3.8 Check your progress

3.1 Introduction:

Linear and quadratic equations are fundamental concepts in algebra that form the backbone of mathematical problem-solving. A linear equation represents a straight-line relationship between variables and is expressed in the form $ax+b=0$, where a and b are constants. It models simple, direct relationships and is widely used in fields like physics, economics, and engineering. Quadratic equations, on the other hand, involve squared terms and are expressed in the standard form $ax^2+bx+c=0$, where a, b and c are constants, and $a\neq0$.

Quadratics represent more complex relationships, often resulting in parabolic graphs, and are crucial in understanding motion, optimization, and geometric problems. Together, linear and quadratic equations provide essential tools for analyzing and interpreting a wide range of mathematical and real-world situations, fostering critical thinking and problem-solving skills.

3.2 Unit objectives

Upon completion of this chapter, you will be capable of

- Recognize linear equations from a set of equations.
- Provide examples of linear equations.
- Formulate a linear equation in one variable and solve it.
- Offer examples and create linear equations in two variables.
- Plot the graph of a linear equation in two variables.
- Determine the solution to a linear equation with two variables.

3.3 Basics of linear equation: forms, solutions, and Graphical representation

A **linear equation** is an equation of the first degree, meaning it involves variables raised only to the first power and has no exponents higher than one. Linear equations describe straight lines when graphed on a coordinate plane. These equations can be in one or more variables, with the most common being equations in one variable or two variables. The general form of a linear equation in one variable is:

$ax+ b = 0$

where a,b are constants and x is the variable.

Types of linear equations:

a) Linear equations in one variable
   A linear equation of one variable has the form
   $ax+bx+c = 0$
   Where a is a non-zero constant and b is another constant.

Steps to solve a linear equation in one variable:

1. **Isolate the variable**: Get the variable (e.g., x) by itself ~~on~~ one side of the equation.

2. **Simplify**: Combine like terms, if necessary.

3. **Solve for the variable**: Perform arithmetic operations (addition, subtraction, multiplication, or division) to find the value of x.

   Example:

   Solving the equation $3x – 5 =10$

   Step1 : add 5 to both L.H.S and R.H.S

   $3x =15$

   Step2: divide both sides by 3

   $X = 5$

   (b)linear equations in two variables.

   A linear equations in two variables is written in the form.

   $Ax +By +C = 0$,

Where A,B,C are constants and x and y are variables.

In two variables, the equation represents a straight line when plotted on the xy-plane.

Example:

$2x + 3y = 6$

This equations represents a straight line. The goal is to find out the relationships between x and y.

Steps to solve:

Solve for one variable in terms of the other (e.g., express y in terms of x).

Use methods like **substitution** or **elimination** to solve if multiple equations are involved.

For instance $2x + 3y = 6$ to y:

$3y = 6 – 2x$

$Y= 2 – 2/3x$

Methods for Solving Systems:

(i)Graphical Method:

Graph each equation on the same coordinate plane.

The point(s) where the lines intersect are the solution(s).

(ii)Substitution Method:

Solve one of the equations for one variable.

Substitute this expression into the other equations.

Solve for the remaining variable(s).

(iii)Elimination Method:

Add or subtract the equations to eliminate one of the variables.

Solve for the remaining variable.

Substitute back to find the other variables.

How we can identify linear equations in one variable and two variable. The following examples are shown in a table

Equations	Linear or non linear
$Y = 2x - 9$	linear
$Y = x^2 - 12$	Non-linear the degree of the variable is two
$\sqrt{y} + x = 6$	Non-linear the degree of the variable y is 1/2
$Y + 4x - 1 = 0$	linear
$Y^2 - x = 10$	Non-linear the degree of the variable y is two.

Linear equations formula:

linear equations formula is a way of representing a linear equation. For example, a linear equations can be written in the standard form, the slope intercept form or the point slope form. Now, if we take the standard form of a linear equations it varies from case to case depending upon the number of variables and it should be remembered that the highest and the only degree of variable of the equation should be 1.

The slope intercept form of a linear equation is written as $y = mx + c$

Where, m= slope and c =y – intercept

slope of a linear equation is given by, $y - y_1 = m(x - x_1)$ is a point on the line.

Note:

The slope of a linear equation is a amount by which the line is rising or falling. It is being calculated by the formula i.e. $(x_1, y_1)(x_2, y_2)$ are any two points on a line then its slope is calculated using the formula

$$\frac{y2 - y1}{x2 - x1}$$

Standard form of Linear equation:

Standard form of a linear equation in one variable is written as: Ax+B=0 where Aand B are real numbers, and x is the only variable is the constant.

Linear equations graph.

If a linear equation involves only the variable y , its graph will appear as a horizontal line parallel to the x -axis. Let's explore how to graph such an equation with a few examples. Example : plot a graph for a linear equations in two variables, x – 2y = 2

Step 1: the given linear equations is x – 2y = 2

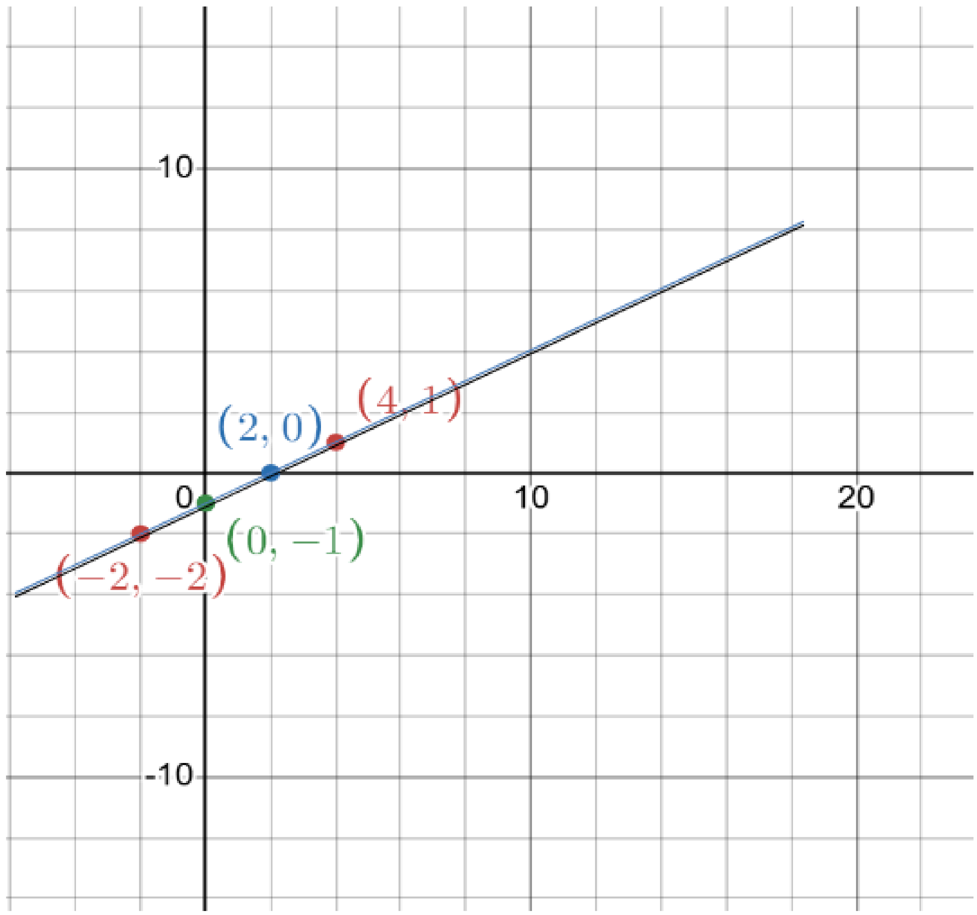
Step 2: now we can replace the value of x for different numbers and get the resulting value of y to create the coordinates.

Step 3: when we put x = 0; in the equation we get y = $\frac{0}{2} - 1$, i.e. y=-1. Similarly, if we substitute the value of x as 2 in equation, y = $\frac{x}{2} - 1$, we get y =0

Step 4: y = 1 is obtained by replacing x with the value 4. Y = -2 is the value that results from x = -2. now these pairs of values of(x,y)satisfy the given linear equation y = $\frac{x}{2} - 1$. Therefore we list the co-ordinates as shown in the following table.

x	0	2	4	-2
y	-1	0	1	-2

Step 6: plotting these points (4,1), (2,0), (0,-1) and (-2,-2) on a graph and joining the points to get a straight line. These is how a linear equations is represented on a graph.



3.3 linear equations applications:

Linear Equations solution in One Variable

Solving a linear equation, it is essential to keep both sides of the equation balanced. The equality symbol signifies that the expressions on either side of it are equivalent. Q). solve (2x - 10)/2 = 3(x -1)

Step1: clear the fraction

x-5 = 3x – 3

step2: simplify both sides equations

x – 5 = 3x -3

$x = 3x + 2$

step 3: isolate x

$x - 3x = 2$

$-2x = 2$

$X = -1$

Solving linear equations.

Solve $x = 12(x + 2)$

Solution:

$x = 12(x + 2)$

$x = 12x + 24$

Subtract 24 on both sides of equation

$x – 24 = 12x + 24 – 24$

$x – 24 = 12x$

Simplify

$11x = -24$

Isolate x:

$x = -24/11$.

Example 2:

Solve $x – y = 12$ and $2x + y = 22$

Solution:

$x = 12(x + 2)$

$x = 12x + 24$

Subtract 24 on both sides of equation

$x – 24 = 12x + 24 – 24$

$x - 24 = 12x$

Simplify

$11x = -24$

Isolate x:

$x = -24/11$.

Example 2:

Solve $x - y = 12$ and $2x + y = 22$

Solution:

Name the equations

$x - y = 12$ …(1)

$2x + y = 22$ …(2)

Isolate Equation (1) for x,

$x = y + 12$

Substitute $x = y + 12$ in equation (2)

$2(y+12) + y = 22$

$3y + 24 = 22$

$3y = -2$

or $y = -2/3$

put the value of y in $x = y + 12$

$x = y + 12$

$x = -2/3 + 12$

$x = 34/3$

Answer: $x = 34/3$ and $y = -2/3$

Solve $5 + x = 8$

Solution: subtracting 5 from both sides of the equation. We get $5 + x - 5 = 8 - 5$

Or $\qquad x + 0 = 3$

Or $\qquad x = 3$

So, x=3 is the solution of the given equation.

Check: when $x = 3$, L.H.S $= 5 + x = 8$ and R.H.S $= 8$

Therefore L.H.S = R.H.S.

Practice Questions

Solve the following linear equations:

1. $5y-11=3y+9$
2. $3x + 4 = 7 - 2x$
3. $9 - 2(y - 5) = y + 10$
4. $5(x - 1) = 3(2x - 5) - (1 - 3x)$
5. $2(y - 1) - 6y = 10 - 2(y - 4)$
6. $y/3 - (y - 2)/2 = 7/3$
7. $(y - 3)/4 + (y - 1)/5 - (y - 2)/3 = 1$
8. $(3x - 2)/3 + (2x + 3)/3 = (x + 7)/6$

Linear equations in two variables.

Q. Neha visited the market to purchase pencils and pens. Each pencil costs Rs. 2, while each pen costs Rs. 4. If she spent a total of Rs. 50, how many pencils and pens did she buy?

Solution: Since we need to determine the no. of pencils and pens Neha bought, let us assume she purchased (x) pencils and (y) pens. Then, Cost of x pencils = Rs.2x

Cost of y pens = Rs.4y

Since total cost in Rs.50, we have

2x + 4y =50 ...(1)

1. If x =1, y =12, then L.H.S = $2 \times 1 + 4 \times 12 = 2 + 48 = 50 \quad and$ R.H.S = 50. Therefore x=1 and y=12 is a solution.
2. If x =3, y=11, then L.H.S= $2 \times 3 + 4 \times 11 = 50$ and RHS= 50, therefore, x= 3, y =11 is also a solution.
3. If x = 4, y= 10, then LHS=$9 \times 4 + 4 \times 10$ = 48 and RHS = 50. Therefore, x=4, y=10 is **not** a solution of the equation.

As a result, a linear equation in two variables has several solutions.

A linear equation in one variable, "x," has the form ax + b=0,a≠0, as we have seen. There is only one solution to it, which is x = (-b)/a. However, the form of a linear equation in two variables, x and y, is ax + by + c = 0.(1)

where at least one of an or b is non-zero and a, b, and c are constants. If a = 0, then (1) can be expressed as follows: ax = -by - c or x = -b/ay - c/a. At this point, we obtain a distinct value of x for every value of y. There will therefore be an endless number of solutions to a linear equation in two variables.

A linear equation with ax + c = 0 and a≠0 can be thought of as

3.4 Introduction to quadratic equation:

A quadratic equation is a polynomial equation of the second degree, which has the general form: $ax^2+bx+c=0$

Where:

- x represents the variable,

In the quadratic equation, a, b, and c are constants, with $a \neq 0$.

Key Concepts of Quadratic Equations:

- **Standard Form**a is the coefficient of x^2 and it must not be zero.
- b is the coefficient of x.
- c is the constant term.

Roots or Solutions: One possible quadratic equation is:

- Two real roots (when the discriminant $b^2-4ac>0$),
- One real root (when the discriminant $b^2-4ac=0$
- No real roots (when the discriminant $b^2-4ac<0$

1. **Discriminant**: The discriminant is the part of the quadratic formula under the square root:

$$\Delta=b^2-4ac$$

2. **Factoring**: Some quadratics can be factored into two binomials.

For example:

$X^2+5x+6= (x+2)\ (x+3)$

$x+2=0$or $x+3=0$

Giving the solutions $x=-2$ and $x=-3$.

3. **Quadratic Formula**: The standard method to determine the roots of a quadratic equation is by applying the quadratic formula:

$$X = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

This formula gives the roots directly, regardless of whether the quadratic can be factored.

5. Completing the Square: This technique rewrites a quadratic equation so that it is easier to solve by taking the form (x−p)2=q. The quadratic formula can also be derived using this method.

Example 1: Solving by Factoring

Solve the quadratic equation:

$x^2-5x+6=0$

Step 1: Factor the quadratic expression.

$x^2-5x+6= (x-2)(x-3)$

Step 2: Set each factor equal to zero.

$x-2=0$,$x-3=0$

Step 3: Solve for x

x=2or x=3

Thus, the solutions are x=2 and x=3

Example 2: Solve by applying Quadratic Formula

Solve the quadratic equation:

$2x^2-4x-6=0$

Step 1: Identify a, b, and c

Here, a=2, b=−4, and c=−6

Step 2: Use the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$$

$$X = \ = \frac{4 \pm \sqrt{16+48}}{4}$$

$$X = \frac{4 \pm \sqrt{64}}{4}$$

$$X = \frac{4 \pm 8}{4}$$

Step 3: solving for the two possible values of x

$$X= \frac{4+8}{4} = 3$$

$$X = \frac{4-8}{4} = -1$$

Thus the solutions are x =3 and x=-1

Q. Solve $2x^2$ - 5x + 3 using quadratic equation formula.

Solution: formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The determinant or b^2-4ac = (-5)2 - 4 × 3 × 2 = 25 - 24 = 1

$\sqrt{b^2 - 4ac}$ = 1

Therefore x = $\frac{-(-5)\pm 1}{2\times 2}$

X= $\frac{5+1}{4}$ = $\frac{6}{4}$ = $\frac{3}{2}$ or x = $\frac{5-1}{4}$ = 1

Thus, the roots are 3/2 and 1.

$2x^2 - 5x + 3 = 0$.

Solution: Given,

$2x^2 - 5x + 3 = 0$

$2x^2 - 2x - 3x + 3 = 0$

2x(x-1)-3(x-1) = 0

(2x-3) (x-1) = 0

So,

2x-3 = 0; x = 3/2

(x-1) = 0; x=1

Consequently, the roots of the given equation are 3/2 and 1.

Q. The quadratic equation must be solved. $2x^2 + x - 300 = 0$ using factorization method.

Solution: $2x^2 + x - 300 = 0$

$2x^2 - 24x + 25x - 300 = 0$

2x (x – 12) + 25 (x – 12) = 0

(x – 12)(2x + 25) = 0

So,

x-12=0; x=12

(2x+25) = 0; x=-25/2 = -12.5

Therefore, 12 and -12.5 are two roots of the given equation.

Q. Solve the quadratic equation 2x2 + x – 528 = 0, using quadratic formula**.**

Solution :

Solution: If we compare it with standard equation, $ax^2+bx+c = 0$

a=2, b=1 and c=-528

Hence, by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now putting the values of a,b and c

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 2 \times -528}}{2 \times 2}$$

X = $\frac{-1\pm\sqrt{4225}}{4}$ x= -1 $\pm \frac{65}{4}$

X = 64/4 or x = -66/4

Therefore, x = 16 and x = -33/2

Q. Find the roots of x2 + 4x + 5 = 0, if any exist, using quadratic formula.

Solution: To determine if a quadratic equation has real roots, we calculate the discriminant value.

D = b^2-4ac = 4^2– 4(1)(5) = 16-20 = -4

Since the square root of -4 does not result in a real number, the given equation has no real roots.

Formation of a quadratic equation from its roots:

To find the standard form of a quadratic equation when the roots are given:

Assume that the quadratic equation ax2+bx+c=0 has roots α and β. Next,

$(x-\alpha)(x-\beta)=0$

On expanding, we get,

$x^2-(\alpha+\beta)x+\alpha\beta=0$, which is the standard form of the quadratic equation.

Here, $a=1, b=-(\alpha+\beta)$ and $c=\alpha\beta$.

Example: Form the quadratic equation if the roots are −3 and 4.

Solution: Given -3 and 4 be the roots of the quadratic equation.

Sum of roots = -3 + 4 = 1

Product of the roots = (-3).(4) = -12

As we know, the standard form of a Quadratic equation is:

x^2 − (sum of roots)x + (product of roots) = 0

Therefore, by putting the values, we get

$x^2 - x - 12 = 0$

Sum and Product of Roots of a Quadratic Equation

Let α and β represent the quadratic equation's roots $ax^2+bx+c=0$. Then,

Sum of roots = $\alpha + \beta = -b/a$

Product of roots = $\alpha\beta = c/a$

Example: Given, $x^2 - 5x + 8 = 0$ is the quadratic equation. Find the sum and product of its roots.

Solution: $x^2 - 5x + 8 = 0$ is the quadratic equation given in the form of $ax^2 + bx + c = 0$.
Hence,
a = 1
b = -5
c = 8

Sum of roots = -b/a = 5

Product of roots = c/a = 8

Unit 3.5 : Solving and applying quadratic equation

Let's look at the quadratic formula. $2x^2 – 5x + 3 = 0$. To solve it, we split the middle term by finding two numbers (-2 and -3) such that their sum is equal to the coefficient of x and their product is equal to the product of the coefficient of x^2 and the constant. So, (-2) + (-3) = (-5) and (-2) × (-3) = 6.

By splitting the middle term, we rewrite the equation as $2x^2 – 2x – 3x + 3 = 0$. Then, we factorize it as $2x(x – 1) – 3(x – 1) = 0$, which further simplifies to $(x – 1)(2x – 3) = 0$. Thus, x = 1 and x = 3/2 are the roots of the given quadratic equation. This method of solving a quadratic equation is called the factorization method.

Exercise

Q1. Find the roots of the following quadratic equation by factorization method.

i) $X^2 – 3x – 10$ ii) 2x2 – x – 6 = 0 iii) $\frac{2x2- x+1}{8}$

iv) $\sqrt{(2)}x^2$ + 7x + 5$\sqrt{2}$ v) $100x^2 – 20x + 1$

i) $X^2 – 3x – 10$

solution :

$x^2 – 5x + 2x – 10 = 0$

$x(x – 5) + 2(x – 5) = 0$

$(x - 5)(x + 2)= 0$

$X – 5 = 0$ and $x + 2 = 0$

X=5 and x = -2

Therefore roots are 5 and -2

ii) $2x^2 + x – 6 = 0$

$2x^2 – 4x – 3x – 6 = 0$

$2x(x + 2) – 3 (x + 2)= 0$

$(2x - 3)(x + 2) = 0$

$2x – 3 = 0$, or $x+2=0$

2x = 3   and x = -2

X = 3/2, or x = -2

Therefore the roots are 3/2 and – 2

iii) $\sqrt{(2)}x^2$ + 7x + 5$\sqrt{2}$

$\sqrt{(2)}x^2 + 5x + 2x + 5\sqrt{2} \ =0$

$\sqrt{(2)}x^2$ + 5x + $\sqrt{(2)}$ $\times \sqrt{(2)}$x + 5$\sqrt{2}$ = 0

X($\sqrt{(2)}$x + 5) + $\sqrt{(2)}$ ($\sqrt{(2)}$x + 5)= 0

($\times \sqrt{(2)}$ + 5)(x +$\sqrt{2}$)

($\sqrt{(2)}$x + 5) = 0 and (x +$\sqrt{2}$) = 0

$\sqrt{(2)}$x = - 5   and x = -$\sqrt{2}$

X = $\frac{-5}{\sqrt{(2)}}$ and x = -$\sqrt{2}$

iv) $2x^2 - x + 1/8 = 0$

multiplying both sides of the equation by 8

$2(8)x^2 - 8(x) + (8)*(\frac{1}{8}) = (0)8$

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

4x(4x - 1) -1(4x - 1) = 0

(4x -1)(4x -1) = 0

4x -1 = 0   and 4x -1 = 0

X = ¼ , ¼

Therefore the roots are ¼ and ¼

V) solution

$100x^2 + 2x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1)(10x - 1) = 0$

$10x = 1$ or $10x = 1$

$X = 1/10$ and $x = 1/10$

Therefore the roots are 1/10 and 1/10

Q2. Solve the problem given in the example.

1) Mahesh and Mohanty together have 45 marbles. After both lose 5 marbles each, the product of the number of marbles they now have is 124. Determine how many marbles they each originally had.

2) A cottage industry produces a specific number of toys per day. The production cost of each toy (in rupees) is calculated as 55 minus the number of toys produced in a day. On a particular day, the total production cost amounted to Rs. 750. Determine the number of toys produced on that day.

Solution 1)

let the no. of marbles Mahesh had be x.

The number of marbles Mohanty had will be(total marbles MINUS the number of marbles Mahesh had) = $45 - x$

(i)both of them lost 5 marbles each:

Mahesh = $x - 5$

Mohanty = $45 - x - 5 = 40 - x$

(ii) product of current numbers of marbles = 124

$(x - 5)(40 - x) = 124$

Solution :$(x - 5)(40 - x) = 124$

$40x - x^2 - 200 + 5x = 124$

$-X^2 + 45x = 200 - 124 = 0$

$-x^2 + 45x - 200 + 5x = 124 = 0$

$-x^2 + 45x – 324 = 0$

$X^2 – 45x + 324 = 0$

$X^2 – 36x – 9x + 324 = 0$

$X(x - 36) - 9(x - 36) = 0$

$(x - 36)(x - 9) = 0$

$X – 36 = 0$ and $x – 9 = 0$

$X = 36 \quad x = 9$

Therefore Mahesh and Mohanty has 36 and 9 marbles.

Solution 2)

Let the number of toys produced in a day be x.

(i)cost of each toy = (55 - x)rupees

(ii)total cost of production = cost of each toy × total number of toys

=> $(55 – x)(x) = 750$.

Solution:

$(55 - x)(x) = 750$.

$55x – x^2 = 750$

$X^2 – 55x + 750 = 0$

$X^2 – 25x – 30x + 750 = 0$

$X(x - 25) – 30(x - 25) = 0$

$(x - 25)(x - 30) = 0$

$X – 25 = 0 \quad x – 30 = 0$

$X = 25 \quad x = 30$

The number of toys produced on that day is either 25 or 30.

Q2. Use the quadratic formula to find the roots of the quadratic equation.

If the given quadratic equationis :$ax^2 + bx + c = 0$, then:

If $b^2 - 4ac \geq 0$ then the roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $b^2 - 4ac < 0$ then no real roots exists.

i)$2x^2 - 7x + 3 = 0$

solution:

a = 2, b= -7, c = 3

$b^2 - 4ac = (-7)^2 - 4\ (2)(3)$

= 49- 24

$b^2 - 4ac = 25 > 0$

Therefore roots are x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$X = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-2)(3)}}{2(2)}$

$X = \frac{-(-7) \pm \sqrt{49 - 24}}{2(2)}$

X= $\frac{7 \pm 5}{4}$ x= $\frac{7+5}{4}$, x = $\frac{7-5}{4}$

X= $\frac{12}{4}$ x = $\frac{2}{4}$

X=3 and x= ½

Therefore the roots are 3, ½.

(ii)$2x^2 + x - 4 = 0$

Solution:

a=2,b =1 and c = -4

we know $b^2 - 4ac = (1^2)-4(2)(-4)$

= 1 + 32

$= 33 > 0$

Therefore Roots are x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-1 \pm \sqrt{33}}{4}$

X = $\frac{-1+\sqrt{33}}{4}$, x = $\frac{-1-\sqrt{33}}{4}$

Solve yourself.

(iii) $4x^2 + 4\sqrt{(3)}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Q4. Three years ago and five years from now, the reciprocal of Rehman's age (in years) equals one-third. Determine his current age.

Solution:

Sum of reciprocal of Rehmans age(in years)3 years ago and 5 years from now is 1/3

Let the present age of Rehman's be x years.

3 years ago, Rehman's age was = x – 3

5 years from now age will be = x+ 5

Using this information and the given condition, we can form the following equation:

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

Solution:

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$

$\frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

$(2x+2)(3) = x^2 + 2x - 15$

$6x + 6 = x^2 + 2x - 15$

$x^2 + 2x - 15 = 6x + 6$

$x^2 + 2x - 15 - 6x + 6 = 0$

$x^2 + 2x - 15 - 6x - 6 = 0$

$x^2 - 4x - 21 = 0$

Finding roots by factorization:

$$x^2 - 7x + 3x - 21 = 0$$

X(x-7)+3x-21=0

X(x-7) +3(x- 7) =0

(x -7)(x+3)=0

X -7 =0   x+3=0

X=7, x=-3

Age can't be negative value.

Therefore Rehman's present age is 7 year.

Q. Shefali received a total score of 30 in both English and mathematics for a class assignment. Her [52] total score would have been 210 if she had received three less English points and two more math points. Look up her grades for both subjects.

Solution:

(i) Shefali's English and math grades add up to thirty.

(ii) Her total score would have been 210 if she had received two more points in math and three less in English.

Assume that Shefali received a math grade of x.

(i) Following that, her English score = 30 - her math score = 30-x

(ii) 2 more math points = x+2 (iii)3marks less in English = 30 – x – 3

=27 – x

The result of these two =210

(x+2)(27-x)=210

(x+2)(27-x)=210

$27x - x^2 + 54 - 2x = 210$

$-x^2 + 25x + 54 - 2x = 210$

$-x^2 + 25x + 54 = 210$

$-x^2 + 25x + 54 - 210 = 0$

$-x^2 + 25x - 156 = 0$

Multiplying both sides by -1:

$-x^2 + 25x + 156 = 0$

Comparing with $ax^2 + bx + c = 0$

a=1, b =-25, c= 156

$b^2 - 4ac = (-25)^2 - 4(1)(156)$

$= 625 - 624 = 1$

$b^2 - 4ac > 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(156)}}{2(1)}$

$= \frac{-(-25) \pm \sqrt{1}}{2(1)}$

$X = \frac{25+1}{2}$ $x = \frac{25-1}{2}$

X = 13 x = 12

Two possible answers for the given questions:

If Shefali scored 13 marks in mathematics, then mark in English = (30 -13) = 17

If Shefali scored 12 mark in mathematics, then mark in English= (30 - 12) = 18.

Q. The difference between the squares of two numbers is 180. Additionally, the square of the smaller number is 8 times the larger number. Find the two numbers.

Solution:

Let the larger number be x.

Square of the smallest number is= 8xType equation here. $\pm\pm$

Difference of squares of the two numbers is 180.

Square of the larger number – square of smaller number = 180

$X^2 - 8x - 180 = 0$

$X^2 - 18x + 10x - 180 = 0$

$X(x-18) + 10(x-18) = 0$

$X - 18 = 0 \qquad x + 10 = 0$

$X = 18 \qquad\qquad x = -10$

If the larger number is 18, then square of the smaller number = 8×18

Therefore smaller number = $\pm\ \sqrt{8 \times 18}$

$= \pm\ \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$

$= \pm 2 \times 2 \times 3$

$= \pm 12$

If larger number is -10,then square of smaller number = $8 \times (-10) = -80$

Square of any number cannot be negative.

Therefore x = -10 is not applicable.

The numbers are 18, 12(or) 18, -12.

Q. A train travels 360 km at a uniform speed. If the speed had been 5km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution:

Let the speed of the train be s km/hr and the time taken be t hours.

Distance = speed $\times\ time$

$360 = s \times t$

$T = (\frac{360}{s})$

Increase speed of train : s + 5

New time to cover the same distance : t – 1

(s + 5)(t - 1)=360 …(2

St – s + 5t – 5 = 360

$360 - s + 5(\frac{360}{s}) - 5 = 360$

$-s + \frac{1800}{s} - 5 = 0$

$-s^2 + 1800 - 5s = 0$

$S^2 + 5s - 1800 = 0$

Solving the quadratic formula :

Comparing with $ax^2 + bx + c = 0$

A =1, b =5, c= -1800

$B^2 - 4ac = 5^2 - 4(1)(-1800)$

=25 +7200

= 7225>0

Therefore the real roots exists.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$s = \frac{-5 \pm \sqrt{7225}}{2}$

$= \frac{-5 \pm 85}{2}$

$S = \frac{-5+85}{2}$, $s = \frac{-5-85}{2}$

S= 40 s = -45

Speed of the train cannot be a negative value.

Therefore speed of the train is 40km/hr.

Finding the nature of roots of a quadratic equation.

Q1. Find the nature of the roots of the following quadratic equations, if the real roots exists, find them.

(i)$2x^2 – 3x + 5 = 0$ (ii)$2x^2 - 6x + 3 =0$ (iii)$3x^2 - 4\sqrt{(3)}x + 4 =0$

We know that the general form of a quadratic equation is $ax^2 + bx + c = 0$

$b^2 - 4ac$ is called the discriminant of the quadratic equation and we can decide whether the real roots are exist or not based on the value of the discriminant:

(i)two distinct real roots, if $b^2 - 4ac>0$

(ii) two equal real roots, if $b^2 - 4ac = 0$

(iii)no real roots, if $b^2 - 4ac < 0$

(i)

Solution:

$2x^2 – 3x + 5 =0$

Here, a =2 , b = -3, c =5

$b^2 - 4ac = (-3)^2 – 4 (2)(5)$

$= 9 – 40$

$= -31, \quad b^2 - 4ac < 0$

Therefore no real roots.

(ii)$2x^2 – 6x + 3 = 0$

Solution:

a =2, b = -6, c = 3

$b^2 - 4ac = (-6)^2 – 4(2)(3)$

$= 36 - 24$

$= 12,\ b^2 - 4ac > 0$

Therefore two distinct real roots.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)2 - 4(2)(3)}}{2(2)}$

$= \frac{6 \pm \sqrt{12}}{4}$

$X = \frac{6 \pm 2\sqrt{3}}{4}$

$X = \frac{3 \pm \sqrt{3}}{2}$

Roots are $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$.

3.7 Unit summary

Linear and quadratic equations are fundamental concepts in algebra. A linear equation, expressed as $ax+b=0$ where $a \neq 0$ represents a straight line when graphed and has a constant rate of change. These equations can be solved using simple algebraic techniques, and in two-variable cases, they describe a line in 2D space. Systems of linear equations can be addressed through substitution, elimination, or graphical methods. On the other hand, quadratic equations, expressed as $ax^2+bx+c=0$ where $a \neq 0$ produce a parabolic graph and are solved using methods such as factoring, completing the square, or the quadratic formula, x= $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The discriminant ($\Delta = b^2 - 4ac$) determines the nature of the roots: two distinct real roots ($\Delta > 0$), one repeated root ($\Delta = 0$), or complex roots ($\Delta < 0$). Linear equations are used to model scenarios with constant rates of change, while quadratic equations are crucial in describing accelerated motion, optimization problems, and parabolic paths, such as in projectile motion. Together, these equations provide the foundation for solving real-world problems and understanding algebraic relationships.

3.8 Check your progress:

Q1. Find the roots of quadratic equations by factorisation:

(i) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2}=0$

(ii) $100x^2 – 20x + 1 = 0$

Q2.Solve the quadratic equation $2x^2 – 7x + 3 = 0$ by using quadratic formula.

Q3.Find the discriminant of the equation $3x^2– 2x +1/3= 0$ and hence find the nature of its roots. Find them, if they are real.

Q4.In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the original duration of the flight.

Unit 4: Geometry of line and line segment

Structure:

4.1 Introduction

4.2 Unit objectives

4.1 Introduction

Geometry is a branch of mathematics that studies shapes, sizes, and the properties of space. One of the fundamental elements of geometry is the line, which plays a critical role in constructing other geometric figures and shapes. A line, however, is different from a line segment, and understanding both concepts is essential in mastering geometry.

4.2 Unit Objectives:

Upon finishing this unit, you will have the ability to:

- Understanding basic concepts of line, line segments and rays.
- Understanding and applying the distance formula to calculate the distance between two given points.
- Interpret the **midpoint** in both real-world and mathematical contexts
- Understand the concept of the **slope** of a line as the rate of change of y with respect to x, or the steepness of a line.
- Calculate the **slope** of a line given two points on the line, using the slope formula.

4.3 Properties of line and equations of a line

1. What is a Line?

In geometry, a line is a one-dimensional figure that extends infinitely in both directions. It has no thickness and is defined by at least two points. A line is usually depicted as a straight path that continues endlessly in both directions.

Key Properties of a Line:

- **Infinite length**: A line has no endpoints and continues infinitely in both directions. **Straightness**: A line has no curves, bends, or angles. It is straight.
- **Defined by two points**: A line is uniquely determined by any two points on it. These two points give the direction and position of the line.

Example:

- If we have two points, say A(1,2) and B(4,6), the line can be denoted as $AB$where the line passes through both points and continues infinitely in both directions.

2. What is a Line Segment?

A **line segment** is a part of a line that has two endpoints. It is a finite portion of a line with a defined length. Unlike a line, which extends infinitely, There are start and end points for a line segment.

Crucial characteristics of a line segment:

- **Finite length**: A line segment has two definite endpoints, and its length can be measured.
- **Straight**: Like a line, a line segment is also straight.
- **Defined by two endpoints**: The segment is specifically confined between its two endpoints.

Different Line Types

1. **Horizontal Line: This line extends from left to right in a horizontal direction.**
   2. Vertical Lines: These are lines that extend vertically from top to bottom.
   3. Perpendicular lines: Perpendicular refers to two straight lines that intersect at a right angle.
   4. Parallel lines – when two lines never intersect or connect, regardless of how many extensions they receive, these two lines are parallel.

Difference between a line and a line segment

Contrast the line and the line segment. Knowing them enables us to differentiate them and hence locate them in planar shapes.

Line	Line Segment
It is an endlessly long collection of points in both directions.	The shortest distance between two points is referred to as the shortest distance.
Lines are not defined by a beginning or an end.	Each segment of a line has a beginning and an end.
Lines are endless, lengthy, and continuous.	The line segments are brief, and each segment represents a piece of the line.
Line AB is denoted as ←→ABAB↔.	Line segment AB is denoted by ¯¯¯ABAB¯
A line cannot be measured or drawn since it is infinitely long in both directions.	Due to the fact that a line segment has a fixed length, it can be visually drawn and measured.

Example:

A line segment AB between points A(1,2) and B(4,6) is denoted as $\underline{AB}$. The length of the segment can be calculated using distance formula.

Length of $\underline{AB}$ = $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

For the given points A(1,2) , B(4,6), the length of $\underline{AB}$ is:

Length $= \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Collinear Points

Three or more points are said to be **collinear** if they lie on the same straight line. To check if points are collinear, you can calculate the slope between pairs of points; if the slopes are equal, the points are collinear.

Example:

Points A(1,2), B(3,4) and C(5,6) are collinear if the slope A and B is same as the slope of B and C.

4.3.1 slope intercept and point-slope forms

Midpoint of a Line Segment

The **midpoint** of a line segment is the point that divides the segment into two equal parts. The midpoint is calculated by averaging the x-coordinates and y-coordinates of the two endpoints.

Midpoint formula

If the end points of a line segment are A(x_1, y1), and B (x_2,y_2), the midpoint is given by:

M = $\frac{x1+x2}{2}, \frac{y1+y2}{2}$

Example:

If A (1,2) and B(5,6)are the endpoints, the midpoint M is

M=$\frac{1+5}{2}, \frac{2+6}{2}$= (3, 4)

Equation of a Line

The equation of a line in a two-dimensional space can be represented in different forms. The most common forms are:

a. Slope-Intercept Form:

The slope-intercept form of a line is:

y=mx+b

Where:

- M is the slope (rate of change) of the line.
- B is the y-intercept (the point where the line crosses the y-axis).

b. Point-Slope Form:

If a line passes through a point (x_1, y1) and has a slope m, its equation is:

$y - y_1 = m(x - x_1)$

Standard Form:

The standard form of a line is:

$Ax+By = C$

Where A,B and C are constant.

Conclusion

The geometry of line segments is fundamental to understanding many geometric principles. A **line** extends infinitely, while a **line segment** is finite and measurable. Various properties such as parallelism, perpendicularity, and collinearity shape the way we study and use lines. You can use these ideas to investigate more complex mathematical topics, analyze shapes, and solve geometric problems.

Solved Examples

Question 1: With the slope 6 and y-intercept 4, find the equation of a line?

Solution:

Here,

m = 6 and

b = 4

With the formula: $y = mx + b$

The equation of the line will be **$y = 6x + 4$**

Question 2: Write the equation of the line in slope-intercept form where slope is -2 and passing through the point (0, 9).

Solution:
Given,

Slope = m = -2
Point = (x, y) = (0, 9)

Equation of a line in slope-intercept form is:

$y = mx + b$....(i)

According to the given,

$9 = (-2)(0) + b$

$b = 9$

Substituting the value of m and b in equation (i),

$y = -2x + 9$

This is the required equation of a line in slope-intercept form.

Question 3: Find the slope and y-intercept of the equation of line $2x – y + 5 = 0$.

Solution:

Given equation of a line:

$2x – y + 5 = 0$

Thus,

$y = 2x + 5$

This is of the form $y = mx + b$

Here, $m = 2$ and $b = 5$

Therefore, slope = 2 and y-intercept = 5

Q. Determine the slope-intercept form of the line passing through the points (2, 4) and (4, 8).

Solution:

First, find the slope $y - y1 = m(x - x1)$

$m = (y - y1) / (x - x1)$

$m = (8 -4) / (4 - 2) \Rightarrow 4 / 2 \Rightarrow 2$

Next, use one of the points, say (2, 4), and the slope to find b:

$4 = 2(2) + b$

$\Rightarrow b = 0$

Required equation is: **y = 2x**

Q. Convert the equation 2x - 3y = 6 to slope-intercept form.

Solution:

Solve for y:

$- 3y = - 2x + 6$

$y = 2x / 3 - 2$

This is the required equation in slope intercept form.

Q. **What is the y-intercept of the line $y = - 5x + 7$?**

Given Equation,

$y = - 5x + 7$

comparing with, $y = mx + c$

The y-intercept is $c = 7$

Q. Find the slope and y-intercept of the line given by the equation $y = x/2 - 4$.

Solution:

Given Equation,

$y = x/2 - 4$

Comparing with, $y = mx + c$

Slope $(m) = 1/ 2$

Y-intercept $(c) = - 4$

Q. Determine the equation of the line with slope -4 that passes through the point (2, -1).

Solution:

Given points,

$(x1, y1) = (2, -1)$

$m = -4$

Using the point-slope form: $y - y1 = m(x - x1)$

$y + 1 = - 4(x - 2)$

Convert to slope-intercept form: $y = - 4x + 7$

Q. Find the equation of a horizontal and vertical line passing through (4, -2)

Solution:

A horizontal line has a slope of '0'

Horizontal passing through (4, -2)

y = - 2

A vertical line has an undefined slope

Vertical line passing through (4, -2)

x = 4

Some practice questions:

Q1. Write the equation of the line with slope -2 and y-intercept 5.

Q2. Convert 4x - y = 7 to slope-intercept form.

Q3. Find the slope and y-intercept of the line passing through points (2, - 1) and (4, 3).

Q4. Determine the equation of the line passing through (1, 2) and having a slope of 3/2.

Q5. Graph the line x/3 - 4.

Q6. Find the equation of the line that passes through the points (0, 0) and (5, 10).

Q7. Write the equation of a line with an undefined slope passing through (2, -3).

Q8. Determine the slope and y-intercept of 6y+3x=12.

Q9. Find the equation of the line with slope 0.5 that passes through (4, 1).

Q10. Convert the equation 2x + 3y = 9 to slope-intercept form.

X and Y Intercept Formula

X and Y Intercept Formula as the name suggests, is the formula to calculate the intercept of a given straight line. An intercept is defined as the point at which the line or curve intersects the graph's axis. The intercept of a line is the point at which it intersects the x-axis or the y-axis.

When an equation isn't in the form y = mx + b, we determine the intercepts by substituting 0 as necessary and solving for the corresponding variable.

- To find the y-intercept, we set x = 0 and solve for y. This yields the point (0, y).
- To find the x-intercept, we set y = 0 and solve for x. This gives us the point (x, 0).

Intercept Definition in X and Y Intercept

An intercept is defined as the point where the line or curve crosses the axis. If the point is on the x-axis then it is called the x-intercept and if the point is on the y-axis, then it is called the y-intercept.

We generally represent the x-intercept by **a** and the y-intercept by **b.** The equation of the line making a and b intercept on the x and y axis respectively is,

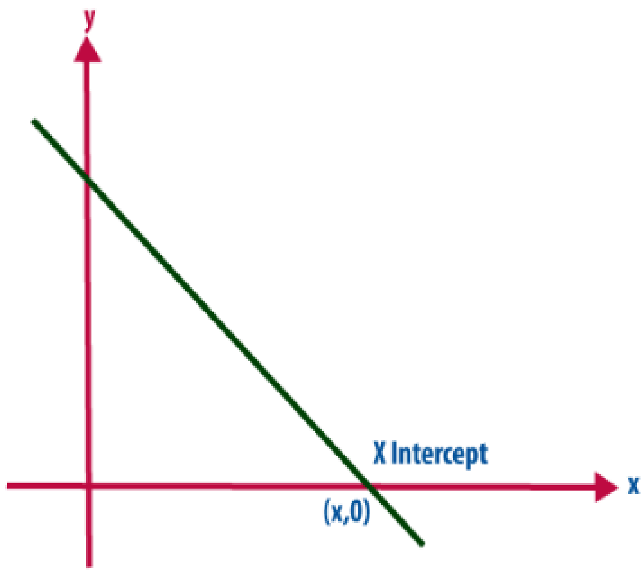
$$\frac{x}{a} + \frac{y}{b} = 1$$

What is X-Intercept?

The x-intercept of a line is the point at which the line intersects the x-axis. So, to find the x-intercept put y = 0 in the equation of a line.

X-Intercept Formula

The formula of x-intercept for slope-intercept equation y = mx + c is given by,



x- intercept = - c/m

Where,

(0, c) is the y-intercept,

m is the slope of the given line

Derivation of X-Intercept Formula

Consider a line given in the slope intercept form y = mx + c, where the line has a intercept (c, 0) and has a slope m.

Put y = 0 in the equation to get the x-intercept.

⇒ 0 = mx + c

Solve the equation for x.

⇒ mx = -c

⇒ x = -c/m

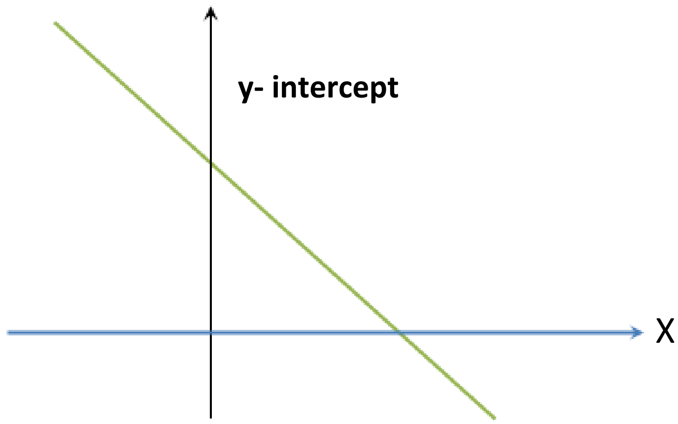
This derives the formula for x-intercept.

What is Y-Intercept?

Y-Intercept Formula

The formula of y-intercept for slope-intercept equation y = mx + c is given by,

Y



Y-intercept = c

Thus, **(0, c)** is the coordinate of y-intercept.

Derivation of Y-Intercept Formula

Consider a line given in the slope-intercept form y = mx + c, where the line passes through the point (0, c) and has a slope m.

Put x = 0 in the equation to get the y-intercept.

⇒ y = m (0) + c

⇒ y = 0 + c

⇒ y = c

This derives the formula for y-intercept.

How To Find X And Y Intercepts?

To find the x-intercept we put y = 0 in the given function and then solve for x. The resultant value of x is the x-intercept of the given function.

Example: Find the x-intercept of the linear equation 2x + 3y = 7.

Solution:

For the x-intercept of the linear equation 2x + 3y = 7

Put y = 0,

2x + 3×0 = 7

⇒ x = 7/2

Thus, the x-intercept of 2x + 3y = 7 is 7/2.

To find the y-intercept we put x = 0 in the given function and then solve for y. The resultant value of y is the y-intercept of the given function.

Example: Find the y-intercept of the linear equation 3x + 4y = 12.

Solution: For the y-intercept of the linear equation 3x + 4y = 12

Put x = 0,

3×0 + 4y = 12

⇒ y = 12/4

⇒ y = 3

Thus, the y-intercept of 3x + 4y = 12 is 3.

Intercept Form of a Straight Line

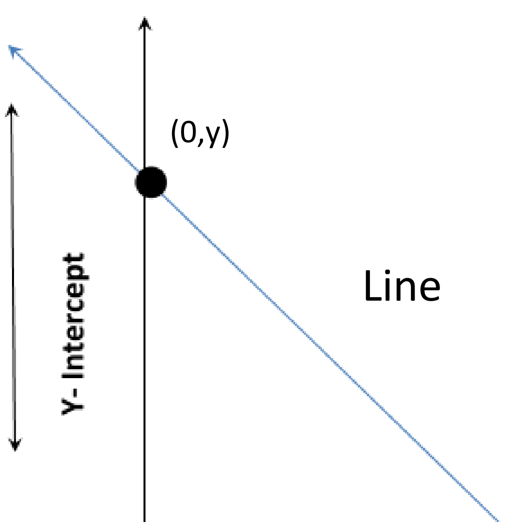
Intercept Form of a Straight Line, mathematically given by

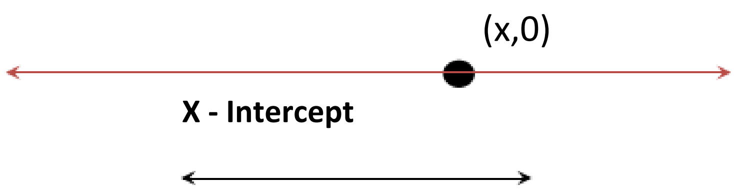
Where, $\frac{x}{a} + \frac{y}{b} = 1$

Where,

a is the x-intercept of the straight line

b is the y-intercept of the straight line





For Point-Slope Form

The point-slope form of a line is given as follows:

$y - \mathbf{y}_1 = m(x - x_1)$

where:

(X_1, Y_1) is a point on the line

m is the slope of the line.

To find, the x and y-intercepts of the given line,

Here, rearranging the equation, we get

y = mx – mx1 + y1

⇒ y = mx + (-mx1 + y1)

Comparing it with y = mx + c, we get

c = -mx1 + y1, which is the y-intercept of the given line.

and x-intercept is -c/m = (mx1 – y1)/m = x1 – y1/m

Thus, x and y-intercept of the given **y – y1 = m(x – x1)** are **x1 – y1/m** and **-mx1 + y1** respectively.

Solved Example of X and Y Intercept Formula

Solution:

We have the equation as, x + 3y = 8.

Put $y = 0$ to find the x-intercept and then solve the equation for x.

$\Rightarrow x + 3(0) = 8$

$\Rightarrow x = 8$

So, the x-intercept for the equation is (8, 0).

Q. Calculate the x-intercept of equation 4x + 7y = 10.

Solution:

We have the equation as, $4x + 7y = 10$.

Put $y = 0$ to find the x-intercept and then solve the equation for x.

$\Rightarrow 4x + 7(0) = 10$

$\Rightarrow 4x = 10$

$\Rightarrow x = 10/4$

$\Rightarrow x = 5/2$

So, the x-intercept for the equation is (5/2, 0).

Q. Calculate the y-intercept of equation 4x + 3y = 24.

Solution:

We have the equation as, $4x + 3y = 24$.

Put $x = 0$ to find the y-intercept and then solve the equation for y.

$\Rightarrow 4(0) + 3y = 24$

$\Rightarrow 3y = 24$

$\Rightarrow y = 24/3$

$\Rightarrow y = 8$

So, the y-intercept for the equation is (0, 8).

Question 1:

Find the distance between the following points:

(I) (-1, 2) and (2, 3)

(II) (0, 1) and (6, –1)

(III) (1, 0, –1) and (2, 0, 7)

Solution:

(I) Let the distance between the points (-1, 2) and (2, 3) be d, then

$d = \sqrt{[(2 - (-1))^2 + (3 - 2)^2]} = \sqrt{[9 + 1]} = \sqrt{10}$ units.

(II) Let the distance between the points (0, 1) and (6, –1) be d, then

$d = \sqrt{[(6 - 0)^2 + (-1 - 1)^2]} = \sqrt{[36 + 4]} = \sqrt{40} = 2\sqrt{10}$ units.

(III) Let the distance between the points (1, 0, –1) and (2, 0, 7) be d, then

$d = \sqrt{[(2 - 1)^2 + (0 - 0)^2 + (7 - (-1))^2]} = \sqrt{[1 + 0 + 64]} = \sqrt{65}$ units.

Question2:

Find a point on the x-axis which is equidistant from the points (–5, 2) and (9, –2).

Solution:

Let the points be P(–5, 2) and Q(9, –2) and let R(x , 0) be the point on x-axis which is equidistant from P and Q.

Now, $PR = QR \Rightarrow PR^2 = QR^2$

$\Rightarrow [(x - (-5))^2 + (0 - 2)^2] = [(x - 9)^2 + (0 - (-2))^2]$

$\Rightarrow (x + 5)^2 + 4 = (x - 9)^2 + 4$

$\Rightarrow x^2 + 10x + 25 = x^2 - 18x + 81$

$\Rightarrow 10x + 18x = 81 - 25$

$\Rightarrow 28x = 56$

$\Rightarrow x = 56/28 = 2$

Therefore, the point is R(2, 0)

Question3. Show that the points P(–1, 6, 6), Q(0, 7, 10) and R(–4, 9, 6) form an isosceles right triangle.

Solution:

Now, PQ = $\sqrt{[(0 + 1)^2 + (7 - 6)^2 + (10 - 6)^2]}$

= $\sqrt{[1 + 1 + 16]} = \sqrt{18} = 3\sqrt{2}$ units.

QR = $\sqrt{[(-4 - 0)^2 + (9 - 7)^2 + (6 - 10)^2]}$

= $\sqrt{[16 + 4 + 16]} = \sqrt{36} = 6$ units.

PR = $\sqrt{[(-4 - (-1))^2 + (9 - 6)^2 + (6 - 6)^2]}$

= $\sqrt{[9 + 9 + 0]} = \sqrt{18} = 3\sqrt{2}$ units.

Also, $PQ^2 + PR^2 = QR^2$. Hence, PQR is a right isosceles triangle.

Collinearity of Points

If A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) are collinear points on a plane, then the sum of any two of these distances is equal to the third distance.

That is, AB + BC = AC

Or, AB + AC = BC

Or, AC + BC = AB

Question4. Show that the points A(2, 3), B(3, 4) and C(4, 5) are collinear.

Solution:

Now, AB = $\sqrt{[(3 - 2)^2 + (4 - 3)^2]}$

= $\sqrt{[1 + 1]} = \sqrt{2}$ units

BC = $\sqrt{[(4 - 3)^2 + (5 - 4)^2]}$

= $\sqrt{[1 + 1]} = \sqrt{2}$ units

AC = $\sqrt{[(4 - 2)^2 + (5 - 3)^2]}$

= $\sqrt{[2^{2.} + 2^2]} = \sqrt{8} = 2\sqrt{2}$ units.

Clearly, AB + BC = AC

∴ the points are collinear.

Practice questions on distance formula.

1. Find the distance between the following points:

(I) (2, 5) and (3, 6)

(II) (–9, 3) and (3, 2)

(III) (4, 5, –2) and (0, –2, 3)

2. Find a point on the y-axis which is equidistant from the points (–5, 2) and (9, –2).

3. Find the area of the square whose one pair of opposite vertices are (2, 6) and (6, 2).

4. Find the orthocentre of the triangle PQR such that P(–1, 6, 6), Q(0, 7, 10) and R(–4, 9, 6).

5. Find the coordinates of the fourth vertex of the parallelogram whose rest of the three vertices are (–2, 3), (6, 7) and (8, 3).

Questions on mid point formula:

Question 1: Find the midpoint of a line whose endpoints are (4, 5) and (6, 7).

Solution:

Given, $(x_1, y_1) = (4, 5)$

$(x_2, y_2) = (6, 7)$

According to the formula we can find the midpoint (x, y):

$(x, y) = [(x_1 + x_2)/2, (y_1 + y_2)/2]$

$(x, y) = [(4 + 6)/2, (5 + 7)/2]$

$= (5, 6)$

Question 2: If (1, 0) is the midpoint of the line joining the points A(-6, -5) and B, then find the coordinates of B.

Solution:

Given,
(1, 0) is the midpoint of A and B.
$A = (-6, -5) = (x_1, y_1)$
Let the coordinates of B are (a, b).

$(a, b) = (x_2, y_2)$
Using the midpoint formula,
$(x, y) = [(x_1 + x_2)/2, (y_1 + y_2)/2]$ $(1, 0) = [(a – 6)/2, (b – 5)/2]$

Now equating the x and y coordinates,
$(a – 6)/2 = 1, (b – 5)/2 = 0$
$a – 6 = 2, b – 5 = 0$
$a = 8, b = 5$
Therefore, the coordinates of B = (8, 5).

Question3: Find the coordinates of the centre of the circle whose endpoints of a diameter are (0, 2), (3, 4).

Solution:

Given coordinates of endpoints of a diameter are:
$(0, 2) = (x_1, y_1)$
$(3, 4) = (x_2, y_2)$
Centre of the circle is the midpoint of diameter.
Coordinates of the centre of a circle = $[(x_1 + x_2)/2, (y_1 + y_2)/2]$

$= [(0 + 3)/2, (2 + 4)/2]$

$= (3/2, 6/2)$

$= (1.5, 3)$.

4.5 Gradient of a Line: calculating and interpreting slope

The gradient of a line, also known as its slope, is a measurement of the line's inclination with respect to the X-axis. It is used to calculate the steepness of a line. Gradient is calculated by the ratio of the rate of change in y-axis to the change in x-axis.

In this article, we will discuss the gradient of a line, methods for its calculation, the gradient of a curve, applications of gradient of a line, some solved examples, and practice problems related to the gradient of a line.

What is Gradient?

Gradient refers to the rate of change of a quantity with respect to some independent variable.

Gradient of a Line Formula

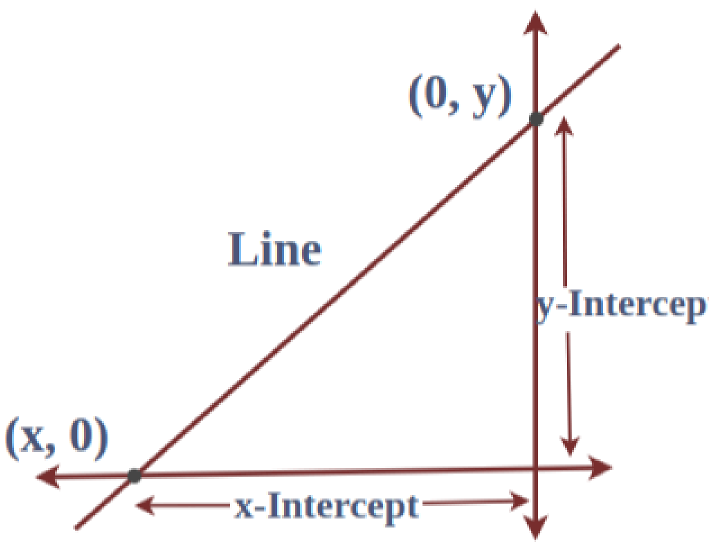
The Gradient of a Line Through Two Points Formula (x1, y1) and (x2, y2) is given by,

m = $(y2 \; - y1\,)/(x2 \; - x1\,)$

OR

m = Δy/Δx

it represents the change in ordinates with respect to change in abscissa for a line. Various methods to calculate gradient or slope of a line are discussed as follows



How to Calculate Gradient of a Line?

There are several ways to determine a line's gradient, or how inclined it is with respect to the X-axis. This degree of inclination is also defined in terms of trigonometric tangent of the angle made by the line with positive X-direction taken in anticlockwise direction.

Gradient of a line is particularly called the slope of a line. Different methods to calculate the gradient of a line based on type of inputs available are discussed as follows:

- Angle of Inclination
- Two points on the line and their coordinates
- Equation of line

Angle of Inclination

Let the angle made by the line with positive X direction taken in anticlockwise direction be θ and m denote the gradient or slope of the line. Then we have,

m = tan θ

Hence, the gradient of a line can be measured by evaluating the value of tangent of the angle made by the line with positive X-direction taken in anticlockwise direction.

Coordinates of Two Points

Let (x1, y1) and (x2, y2) denote the coordinates of two points on the line and m be the slope of the line. Then we have,

m = $(y2 - y1)/(x2 - x1)$

Hence, the other method to calculate the gradient of a line is given the ratio of change in y-coordinate to the change in x-coordinate.

Equation of Line

If the line's equation is expressed as ax + by + c = 0, we express it as y = mx + c, where'm' stands for the line's gradient or slope. The following is how it is expressed mathematically:

We have ax + by + c = 0

⇒ y = (-a/b) x + (-c/b)

⇒ m = -a/b

Thus, for a line represented as ax + by + c=0, slope or gradient is given as m = -a/b, i.e. -(coefficient of x)/(coefficient of y).

Solved Questions on Gradient of a Line

Question 1: Find the gradient of a line which passes through the points (3,5) and (1,4).

Solution:

We are aware that a line's gradient passing through (x1, y1) and (x2, y2) is given as,

m = (y2-y1)/(x2-x1)

Therefore, gradient or slope of the given line would be, m = (5-4)/(3-1) = 1/2

Thus, we have calculated the gradient of given line as 1/2.

Question 2: A line makes an angle of 60° with positive X-direction in anticlockwise direction. Find the gradient of line.

Solution:

Here, we have θ = 60° and we know that,

Gradient of a line, m = tan 60° = √3

Thus, gradient of the given line is found to be √3.

Question 3: What is gradient of the line represented as 3x+4y+5=0?

Solution: We know that,

Slope or Gradient of a line = - (coefficient of x)/ (coefficient of y)

Therefore, for given line, we have,

Gradient, m = -3/4

Question 4: Determine the gradient and nature of the curve represented as y = x2 + 5x + 12 at x=2.

Solution:

For any curve, gradient is the slope of tangent line drawn at the given point and it given as dy/dx. Here we have,

y = x2 + 5x + 12

⇒ dy/dx = 2x + 5

At x = 2, dy/dx = 2 × 2 + 5 = 4 + 5 = 9

4.7 unit summary

The unit on the geometry of lines and line segments delves into the fundamental building blocks of geometric understanding. Lines are infinitely extending straight paths with no endpoints, while line segments are finite portions of a line bounded by two endpoints. This unit explores their properties, such as length, midpoint, and slope, along with their applications in coordinate geometry. Students learn about the relationships between lines, including parallelism, perpendicularity, and angles formed when lines intersect. Practical tools like the distance formula and midpoint formula are introduced, enabling precise calculations in both theoretical and real-world contexts.

4.8 Check your progress

Q1: Determine the gradient of a line that connects (1, 2) and (3, 4).

Q2: Find the angle made by a line with positive direction of X-axis whose gradient is 1/√3.

Q3: What is the gradient, or slope, of the line that the equation 4x+3y+12=0 represents?

UNIT 5: Profit and loss and interest computation

5.1 Introduction:

Profit and Loss is a fundamental concept in business and finance that plays a significant role in competitive examinations, especially in quantitative aptitude sections. It deals with the financial aspects of transactions involving buying and selling goods.

1. Basic definition:

- Cost **Price (CP):** The price at which an item is purchased or acquired.
- **Selling Price (SP):** The price at which an item is sold to a customer.
- **Profit:** The positive difference between the selling price and the cost price.

  Profit = Selling price (SP) – cost price(CP)

  Loss: the negative difference between the cost price and selling price. It can be written as:

  Loss = cost price (CP) – selling price (SP)

2. **Key basic formulas :**

   When profit is made
   SP = CP + profit

   When loss is incurred
   SP= CP - loss

   If the profit percentage is given

   $$SP = CP\left(1 + \frac{profit\ percentage}{100}\right)$$

If the loss percentage is given

SP = CP (1 - $\frac{loss\ pecentage}{100}$)

Some examples to understand the concept better

5.2 Unit objectives

Unit Objectives:

By the end of this unit, you will be able to:

- Specify the cost price, selling price, profit, loss, and marked price.
- Solve problems to calculate profit or loss using appropriate formulas.
- Compute profit and loss percentages and understand their implications.
- Apply Concepts to Real-Life Scenarios

5.3 Fundamentals of profit and loss calculations

Profit calculation:

Example 1. Profit calculation

Q. A shopkeeper buys television for Rs. 25000 and sells it for Rs. 28000. Calculate the profit percentage.

Solution : given,

Cost price(CP) = Rs. 25000

Selling price(SP) = Rs. 28000

Profit = SP – CP = 28000 – 25000 = Rs. 3000

Clearly we can see the profit is made

Therefore, profit percentage = $\frac{3000}{25000} \times 100$ = 12%

So, the profit percentage is 12%

Example2. Loss calculation

Q. a person sells a scooter for Rs 15000,which he bought for Rs.20000.what is his loss percentage?

Solution: Cost Price (CP) = ₹20,000

Selling Price (SP) = ₹15,000

The loss incurred is:

Loss=CP−SP=20,000−15,000=₹5,000

Loss percentage is:

Loss Percentage=$\frac{5000}{2000}$×100= 25%

So, the loss percentage is **25%**.

Successive profit or loss

When multiple successive profit or loss percentages are involved, the overall profit or loss is calculated using the formula:

Overall SP = original CP $\times (1 + \frac{P1}{100}) \times (1 + \frac{P2}{100})$

Where P_1, P_2 are successive profit or loss percentage.

Some examples of successive profit.

Q. a person buys a product for Rs. 10000 . He first gain 20% profit and 10% profit on the new price. Find the selling price?

Solution

initial Cost Price (CP) = ₹10,000

First Profit = 20%

New price after 20% profit:

New SP = 10,000×(1+$\frac{20}{100}$) = 10,000×1.2=₹12,000

Second Profit = 10%

New price after 10% profit:

Final SP=12,000×(1+$\frac{10}{100}$) =12,000×1.1=₹13,200

So, the final selling price is **₹13,200**.

Discount and profit/loss

In many competitive exams, the concept of discount is also intertwined with profit and loss. A discount is a reduction in the selling price of an item.

If an item is sold at a discount:

- **Marked Price (MP):** The original price or the price before any discount.
- **Selling Price (SP):** The price after the discount is applied.

The relationship between these is:

SP=MP−Discount

Examples discount and profit

Q. A man buys an item at ₹2,000 and sells it at ₹2,500 after giving a 10% discount on the marked price. Find the profit percentage.

Solution:

Cost Price (CP) = ₹2,000

Selling Price (SP) = ₹2,500

Discount = 10%

Let the marked price be MP. Since the item is sold at a 10% discount:

SP=MP− ($\frac{10}{100}$×MP) =0.9×MP

Thus,

2,500=0.9×MP $\Longrightarrow$ MP=$\frac{2500}{0.9}$= ₹ 2,777.78

Profit made is:

Profit=SP−CP=2,500−2,000=₹ 5000

Profit percentage:

Profit Percentage=$\frac{500}{2000}$×100=25%

5.4 Applications of profit and loss.

Q.1 A vendor bought 6 oranges for Re 10 and sold them at 4 for Re 6. Find his loss or gain percent.

Solution:

Suppose, number of oranges bought = LCM of 6 and 4 = 12

∴CP = Re (10/6 × 12) = Re 20 and SP = Re (6/4 × 12) = Re 18

∴Loss% = (2/20 × 100)% = 10%

Q.2 By selling 33 meters of cloth, one gains the selling price of 11 meters. Find the gain percent**.**

Solution: (SP of 33m) - (CP of 33m) = Gain = SP of 11m

∴ SP of 22m = CP of 33m

Let CP of each meter be Re 1. Then, CP of 22m = Re 22.

Hence SP of 22m = Re 33.

∴ %Gain = 11/22 * 100

=> 50%

Q3. Pure ghee costs Re 100 per kg. A shopkeeper mixes vegetable oil costing Re 50 per kg and sells the mixture at Re 96 per kg, making a profit of 20%. In what ratio does he mix the pure ghee with the vegetable oil.

Solution: Mean Cost price = Re (100/120) × 96 = Re 80 per kg

Apply rule of allegation,

Therefore, Required ratio = 30:20 = 3:2

Q.4.The CP of 25 articles is equal to SP of 20 articles. Find the loss or gain percent.

Solution:

Let the CP of each article = Re 1.

Then CP of 20 articles = Re 20.

SP of 20 articles = CP of 25 articles = Re 25.

$\therefore$ Gain% = (5/20) × 100% = 25%

Q.5 Anil buys a calculator for Re 600 and sells it to Vikash at 10% profit. Vikash sells it to Chandan for 5 % profit. Chandan after using it for certain time, sells it to Dinesh at a loss of 20%. For how much Chandan sell the calculator to Dinesh.

Solution:

SP for Chandan = 600 × (110/100) × (105/100) × (80/100)
= 600 × 924/1000
= Re 554.40

Q.6 An article is sold by X to Y at a loss of 20%, Y to Z at a gain of 15%, Z to W at a loss of 5% and W to V at a profit of 10%. If v had to pay Re 500, how much X paid for it?

Solution :

CP for X = 500 × (100/80) (100/115) × (100/95) × (100/110)
= 500 × 10000/9614
= Re 520.07

Q.7. A trader marks the SP of an object at a profit of 20%. Considering the demand of the object, he further increases the price by 10%. Find the final profit %.

Solution:
Let the CP = Re 100
$\therefore$ SP = 100 × (120/100) × (110/100)
= Re 132

Final profit = (132 - 100) × 100%
= 32%

Q. An article when sold for Re 4600 makes a 15% profit. Find the profit or loss % if it was sold for Re 3600.

Solution: CP = 4600 $\times$ (100/115)

=Rs. 4000

=> Loss% = [(4000-3600)/4000] $\times$ 100%

= 10%

Practice questions:

Q1. When a man sold an article for Re 540, he made a loss of 10%. At what price should he sell it, so that he incurs a loss of only 5%.

Q2. Ram sells chocolates at a profit of 20% for Re 60. What will be the percentage loss or gain if he reduces the price to Re 55 due to less demand.

Q.3. A 10% hike in the price of wheat forces a person to purchase 2 kg less for Re 110. Find the new and the original price of the wheat.

Q.4 A man sold two plots for Re 8 lakhs each. One on he earns a profit of 16% and the other he loses 16%. How much does he loss or gain in the whole transaction**?**

Q.5 An uneducated retailer marks up all his goods by 50% above the cost price. He believes that with a 25% profit, he will be successful, and thus offers a 25% discount on the marked price. What is the actual profit he makes on the sales?

Q.6 Ruhina bought 20 apples for Rs.150 and sold them at the rate of 15 apples for Rs.150. calculate the percentage of profit made by her?

Q.7 Farhan purchased a house for Rs.80000 and a site for Rs.10000 respectively, if he sold the house for Rs.86000 and the site for Rs.14000, then find the resultant percentage of gain?

Q.8 If the wage of A is 10% more than that of B and the wage of B is 20 % not as much as that of C, than the pay of A,B, C are in the proportion.

Q.9 The aggregate expense cost of two watches is R. 840. One is sold at a benefit of 16% and the other at lost 12%. There is no misfortune or addition in the entire exchange. The expense cost of the watch on which the businessperson additions, is?

Q.10. Raman purchased a fountain at 9/10 of its stamped value and sold it at 8% more than its stamped cost. His increase percent is:

5.5 Simple interest: Concept and calculations

Concept of simple interest.

Introduction: The method of calculating the interest earned or charged on a principal amount of money over time at a fixed rate is known as simple interest, or SI. It is frequently tested in competitive examinations such as the SSC, banking exams (IBPS, SBI), and other government employment tests. The idea is simple and includes fundamental mathematical operations.

1. Key Definitions:

- **Principal (P):** The original sum of money invested or loaned.
- **Rate of Interest (R):** The percentage at which the interest is calculated on the principal amount.
- **Time (T):** The duration for which the money is invested or borrowed, usually expressed in years.
- **Interest (I):** The additional money paid or received for the use of the principal amount, calculated at the given rate and for the specified time period.
- **Amount (A):** The total sum of money after adding the interest to the principal.

  A=P+I

2. Simple Interest Formula:

Simple Interest (SI) is:

I=$\frac{P\times R\times T}{100}$

Where:

I is the Interest.

The principal is P.

The annual rate of interest is denoted by R.

The time period, T, is expressed in years.

3. Formula for Amount (A):

The **Amount (A)** after interest is added to the principal:

$A=P+I= P+ \frac{P\times R\times T}{100}$

On , simplifying:

$A=P\ (1+\frac{R\times T}{100})$

Key Relationships and Points to Remember:

- **Interest is directly proportional to the Principal.**

  If the principal increases, the interest will also increase.

- **Interest is directly proportional to Time.**

  The longer the time, the greater the interest.

- **Interest is directly proportional to the Rate of Interest.**

  The higher the rate, the greater the interest.

- **The formula assumes a constant rate of interest over the entire period of time.**

5. Solved Examples for Better Understanding:

Example 1: Simple Interest Calculation

Question:
For three years, a person invests ₹5,000 at a 6% annual interest rate. After three years, figure out the total amount and simple interest.

Solution:

- Principal (P) = ₹5,000
- Rate of Interest (R) = 6% per annum
- Time (T) = 3 years

Using the formula for Simple Interest:

$I=\frac{P\times R\times T}{100}$

$I = \frac{5000\times 6\times 3}{100} = \frac{90000}{100} =$ ₹900

The **Simple Interest (I)** is ₹900.

To find the **Amount (A)** after 3 years:

A=P+I=5000+900=₹5,900

Answer:
The Simple Interest is ₹900, and the Total Amount after 3 years is ₹5,900.

Example 2: Finding Principal (P) from Simple Interest

Question:
A sum of money earns ₹2,400 as interest in 4 years at the rate of 8% per annum. Find the Principal.

Solution:

Simple Interest (I) = ₹2,400

Rate of Interest (R) = 8% per annum

Time (T) = 4 years

Using the Simple Interest formula:

$I=\frac{P\times R\times T}{100}$

Substitute the given values:

$2400=\frac{P\times 8\times 4}{100}$

$2400 =\frac{32P}{100}$

Multiply both sides by 100:

$2400\times 100 =32P \implies 240{,}000=32P$

Solve for P

P=$\frac{240000}{32}$ = ₹7,500

Answer:
The Principal is ₹7,500.

Example 3: Finding Rate of Interest (R)

Question:
A sum of ₹10,000 earns ₹2,000 as interest in 5 years. Find the Rate of Interest per annum.

Solution:

Principal (P) = ₹10,000

Simple Interest (I) = ₹2,000

Time (T) = 5 years

Using the Simple Interest formula:

I=$\frac{P\times R\times T}{100}$

Substitute the given values:

$2000 = \frac{1000\times R\times 5}{100}$

$2000 = \frac{50000R}{100}$

$2000 \times 100 = 50000R$

Solve for R

$R = \frac{200000}{50000} = 4\%$

Answer:
The Rate of Interest is **4%** per annum.

Example 4: Time Calculation (T)

Question:
A sum of ₹3,600 earns ₹1,080 as interest at the rate of 6% per annum. Find the Time period for which the money is invested.

Solution:

- Principal (P) = ₹3,600
- Simple Interest (I) = ₹1,080
- Rate of Interest (R) = 6% per annum

Using the Simple Interest formula:

I=$\frac{P\times R\times T}{100}$

Substitute the given values:

$1080 = \frac{3600\times 6\times T}{100}$

$1080 = \frac{21600T}{100}$

T = 1080 $\times 100 = 21600$T

Therefore T =$\frac{108000}{21600}$

T = 5year

Answer:
The Time period is **5 years**.

6. Applications of Simple Interest:

- **Loans and Borrowings:** Simple Interest is commonly used to calculate interest on short-term loans (e.g., personal loans, car loans).
- **Investments:** It's also used to calculate the returns on simple investment plans like Fixed Deposits (FDs) or recurring deposits with simple interest.
- **Banking:** Banking exams often include questions on calculating the interest on deposits or loans, using the concept of Simple Interest.

Aptitude questions asked in the examination.

Q1: What would be the annual interest earned on a deposit of Rs. 10,000 in a bank offering a 4% per annum simple interest rate?

Solution. Here P = 10000

R= 4%

T= 1 year

Applying the formula

SI = $\frac{P \times R \times T}{100}$

SI = $\frac{1000 \times 4 \times 1}{100} = 400$

Therefore, the annual interest would be Rs 400

Q2. A certain sum of money amounts to Rs.28000 in 2 years at 30% simple interest per annum. Find the sum.

Solution: here,

A= 28000, T = 2 years, R= 30%

A = P+ SI

A = P + ($\frac{P \times R \times T}{100}$)

A= P [(1 +($\frac{R \times T}{100}$)]

28000 = P [1+ 0.4]

P = $\frac{28000}{1.4}$ P = 20000

Thus the required sum is Rs. 20000

Q3. A man borrows a certain amount of money at varying interest rates: 6% per annum for the first two years, 9% per annum for the next three years, and 14% per annum for the period beyond 5 years. Calculate the initial amount he borrowed if, after nine years, he pays a total interest of Rs. 22,800.

Solution: let the burrowed sum be P

SI for 1st two years + SI for the next 3 years + SI for next 4years= 22800

=> ($\frac{P \times 6 \times 2}{100}$) + ($\frac{P \times 9 \times 3}{100}$) + ($\frac{P \times 14 \times 4}{100}$) = 22800

=> $\frac{95P}{100}$ = 22800

=>P = 24000

Q.4 John takes out a personal loan from his local bank for $15,000 to remodel his kitchen. The bank gives him a 4.5% annual simple interest rate. How much interest will John pay at the end of the loan period if he intends to pay it back in five years?

Solution To find the simple interest, we use the formula:

SI = P × R × T / 100

P = Rs 15,000
R = 4.5%
T = 5 years

SI = 15000 × 4.5 × 5 / 100

So, by the end of the 5 years, John will pay Rs.3,375 in interest on the Rs.15,000 loan.

5.6 Concept of compound interest

The interest charged on a loan or the amount deposited is known as compound interest.It is the idea that we use the most on a daily basis. A sum's compound interest is determined by both principle and interest accrued over a specific time period. This is the primary distinction between simple and compound interest.

Suppose we observe our bank statements, we generally noticed that some interest is credited in our account every year. This interest varies with each year for the same principle amount. We observe that interest increases for successive years. Hence, we can conclude that the interest charged by the bank is not simple interest; this interest is known as compound interest denoted by C.I

Compound interest in math.

Compound interest in mathematics can be computed in a variety of ways depending on the circumstances. The interest formula for compound interest accrued over time can be applied. The following is the formula for compound interest.

Compound interest = Amount – Principal

Here, the **amount** is denoted as :

A = P$\left(1+\frac{R}{100}\right)^{T}$

Where A = amount

P = principal

R = rate of interest

n = no of times interest is compounded per year

t= time(in years).

Alternatively, we can write the formula given below,

CI = A – P and

CI = P $\left(1+\frac{r}{100}\right)^{nt} - P$

This formula is called periodic compound formula.

Here,

- A represents the new principal sum or the total amount of money after compounding period.
- P represents the original amount or principal amount
- The number of times interest is compounded in a year is represented by the symbol n.
- t stands for the number of years.

Interest compounded for different years.

Time(in years)	Amount	interest
1	P(1 + R/100)	$\frac{PR}{100}$
2	$P(1 + \frac{R}{100})^2$	$P(1 + \frac{R}{100})^2$ - P
3	$P(1 + \frac{R}{100})^3$	$P(1 + \frac{R}{100})^3$ - P
4	$P(1 + \frac{R}{100})^4$	$P(1 + \frac{R}{100})^4$- P
n	$P(1 + \frac{R}{100})^n$	$P(1 + \frac{R}{100})^n$ -P

The above formulas help determine the interest and amount in case of compound interest quickly.

solved examples.

Question 1: Akshit borrows Rs. 100,000 for two years at a 10% annual compound interest rate. Determine the amount he must pay at the end of two years, including the compound interest..

Solution :

Given,

Principal/ Sum = Rs. 10000,   Rate = 10%, and Time = 2 years

Let's figure out the second-year amount and interest, which is provided by

Amount (A_2) = $P\left(1+\frac{R}{100}\right)^2$

Putting the values in the above equation

$A_2 = 10000\left(1+\frac{10}{100}\right)^2 = 10000\left(\frac{11}{10}\right)*\left(\frac{11}{10}\right)$ = Rs.12100

Compound interest for 2nd year = $A_2 - P$ = 12100 – 10000 = 2100.

Question 2. **What is the compound interest (CI) on Rs.5000 for 2 years at 10% per annum compounded annually?**

Solution:

Principal (P) = Rs.5000, Time (T)= 2 year, Rate (R) = 10 %

We have amount,

$A = P\left(1+\frac{R}{100}\right)^T$

$A = 5000\left(1+\frac{10}{100}\right)^2$

$A = 5000 \times \frac{11}{10} \times \frac{11}{10}$

= 50 $\times 121 = Rs.6050$

Interest(2nd year) = A – P = 6050 – 500= Rs.1050

Question3. How much compound interest, at 10% annually compounded every six months, must be paid on a loan of Rs. 2000 for three or two years?

Solution : given,

Principal P= Rs.2000

Time, (T) = 2*($\frac{3}{2}$) = 3 years

Rate, R = $\frac{10\%}{2}$= 5%

Amount A can be given as;

A = P $(1 + \frac{R}{100})^t$ => A= $2000(1 + \frac{5}{100})^3$

= 2000 $\times$ $21/20^3$

= 2315.25

CI = A – P = Rs. 2315.25 – Rs. 2000 = Rs. 315.25

Question 4. Find the compound interest (CI) on Rs. 12,600 for 2 years at 10% per annum compounded annually.

Solution: **Solution:**

Given,

Principal (P) = Rs. 12,600

Rate (R) = 10

Number of years (n) = 2

A = $P[1 +(R/100)]^n$

= $12600[1 + (10/100)]^2$

$= 12600[1 + (1/10)]^2$

$= 12600\ [(10 + 1)/10]^2$

$= 12600 \times (11/10) \times (11/10)$

$= 126 \times 121$

$= 15246$

Total amount, A = Rs. 15,246

Compound interest (CI) = A – P

= Rs. 15,246 – Rs. 12,600

= Rs. 2646

Question 5: A refrigerator was purchased for Rs. 21,000. The value of the refrigerator was depreciated by 5% per annum. Find the value of the refrigerator after 3 years. (Depreciation means the reduction of value due to use and age of the item)

Solution:

Principal (P) = Rs. 21,000

Rate of depreciation (R) = 5%

n = 3

Using the formula of CI for depreciation,

$A = P[1 – (R/100)]^n$

A = Rs. $21,000[1\ (5/100)]^3$

= Rs. $21,000[1 – (1/20)]^3$

= Rs. $21,000[(20 – 1)/20]^3$

= Rs. $21,000 \times (19/20) \times (19/20) \times (19/20)$

= Rs. 18,004.875

Therefore, the value of the refrigerator after 3 years = **Rs. 18,004.875.**

Question 7. Find the compound interest on Rs 48,000 for one year at 8% per annum when compounded half-yearly.

Solution :

Given,

Principal (P) = Rs 48,000

Rate (R) = 8% p.a.

Time (n) = 1 year

Also, the interest is compounded half-yearly.

So, $A = P[1 + (R/200)]^{2n}$

$= \text{Rs. } 48000[1 + (8/200)]^{2(1)}$

$= \text{Rs. } 48000[1 + (1/25)]^{2}$

$= \text{Rs. } 48000[(25 + 1)/25]^{2}$

= Rs. 48,000 × (26/25) × (26/25)

= Rs. 76.8 × 26 × 26

= Rs 51,916.80

Therefore, the compound interest = A – P

= Rs (519, 16.80 – 48,000)

= Rs 3,916.80

5.7 Unit summary

The concepts of **Profit and Loss** deal with financial transactions in buying and selling goods or services. Profit arises when the selling price exceeds the cost price, while a loss occurs when the cost price is greater than the selling price. Key formulas include **Profit% = (Profit / Cost Price) × 100** and **Loss% = (Loss / Cost Price) × 100**.

5.8 Check your Progress

1.calculate the compound interest on Rs. 30,000 at 7% per annum is Rs. 4347. The period (in years) is:

2. How much compound interest will be paid on Rs. 25,000 after three years at a 12% annual interest rate?

3.What annual compound interest rate will turn a sum of Rs. 1200 into Rs. 1348.32 in 2 years?

Annuities and Financial mathematics

Structure:

6.1 Introduction

6.2 Unit objectives

6.4 Basics of Annuities: Types and Real-Life Applications

6.5 Calculating Present and Future Value of Annuities

6.1 Introduction:

Financial mathematics is a field that applies mathematical principles and techniques to solve problems in finance. It plays a vital role in understanding and managing financial products, investments, and risk management. When deciding on investments, retirement planning, and other financial objectives, people and organizations must have a solid understanding of annuities and their valuation.

What is an Annuity?

An annuity is a financial contract that provides a series of payments at consistent intervals over a specified period. two types of annuities are:

- **Fixed Annuities**: Provide a constant payment amount over the life of the contract.
- **Variable Annuities**: Payments can vary based on the performance of investments or other factors.

Financial mathematics is essential for evaluating the value of annuities, determining the appropriate investment strategies, and managing the associated risks. The present value (PV) or future value (FV) of the payments, which are computed using interest rates or discount factors, is usually what determines the value of an annuity.

Key concepts in financial mathematics related to annuities include:

1. **Interest Rates and Discounting**: Annuities are valued based on the interest rate or discount rate used to calculate the present value or future value of the payment stream.

2. **Compound Interest**: Most annuities involve compound interest, where the interest earned on an investment is added to the principal, allowing for the growth of the annuity over time.

3. **Actuarial Mathematics**: In actuarial contexts, the study of annuities involves understanding how factors like mortality rates, inflation, and other demographic variables affect the structure and value of the annuity.

6.2 Unit objectives:

Upon completion of this unit on Annuities and Financial Mathematics, students will be able to:

Define an annuity and differentiate between various types of annuities, including fixed, variable, immediate, and due annuities.

- Identify and explain the key features of an annuity, such as payment intervals, duration, and payment amounts.
- Derive and apply the formulas to calculate the present value (PV) and future value (FV) of an ordinary annuity (annuity in arrears) and an annuity due.

- Determine the amount of periodic payments needed for a specific annuity's present or future value.

- Evaluate the advantages and disadvantages of different annuity products (e.g., fixed annuities, variable annuities, immediate vs. deferred annuities).

6.3 Finding Annuity Values in the Present and Future

What is Annuity Formula?

The annuity formula helps in determining the values for annuity payment and annuity due based on the present value of an annuity due, effective interest rate, and several periods. Hence, the formula is based on an ordinary annuity that is calculated based on the present value of an ordinary annuity, effective interest rate, and several periods. The annuity formulas are:

Annuity = r * PVA Ordinary / [1 – (1 + r)$^{-n}$]

Annuity = r * PVA Due / [{1 – (1 + r)$^{-n}$} * (1 + r)]

The annuity formula for the present value of an annuity and the future value of an annuity is very helpful in calculating the value quickly and easily. The Annuity Formulas for future value and present value are:

The future value of an annuity, FV = P× ((1+r) ^{n}−1) / r

The present value of an annuity, PV = P× (1− (1+r)$^{-n}$) / r

Annuity Formula

The present [38] value of the ordinary annuity and the present value of the due annuity are two crucial factors that are used to calculate the formula.

Annuity = r * PVA Ordinary / [1 – (1 + r)$^{-n}$]

Where,

PVA Ordinary = Present value of an ordinary annuity

r = Effective interest rate

n = Number of periods

Annuity = r * PVA $_{Due}$ / [{1 – (1 + r)$^{-n}$} * (1 + r)]

Where,

PVA Due = Present value of an annuity due

r = Effective interest rate

n = number of periods

The Annuity Formulas for future value and present value is:

The future value of an annuity, FV = P×((1+r)^{n}−1) / r

The present value of an annuity, PV = P×(1−(1+r)$^{-n}$) / r

Where,

P = Value of each payment

r = Rate of interest per period in decimal

n = Number of periods

Types of Annuities

Annuities, in this sense of the word, are divided into 2 basic types: ordinary annuities and annuities due.

Annuities Due: With an annuity due, payments, on the contrary come at the start of each time period. Rent, which landlords typically need at the initiation of each month, is one of the common annuity examples.

Examples Using Annuity Formula

Example 1: Dan was getting $100 for 5 years every year at an interest rate of 5%. Find the future value of this annuity at the end of 5 years? Calculate it by using the annuity formula.

Solution

The future value

Given: r = 0.05, 5 years = 5 yearly payments, so n = 5, and P = $100

$FV = P\times ((1+r)^{n} - 1) / r$

$FV = \$100 \times ((1+0.05)^{5} - 1) / 0.05$

FV = 100 × 55.256

FV = $552.56

Therefore, the future value of annuity after the end of 5 years is $552.56.

Example 2.Calculate the future value of the ordinary annuity and the present value of an annuity due where cash flow per period amounts to Rs. 1000 and interest rate is charged at 0.05%?

Solution:

Using the formula to calculate future value of ordinary annuity = $C \times [(1 + i)^{n - 1/i}$

= Rs. 1,000 × $[0.05\ (1 + 0.05)^{5-1}]$

=Rs.1, 000 × 5.53=Rs. 5,525.63

Note that the one-cent difference in these outcomes, Rs. 5,525.64 vs. Rs. 5,525.63, is because of rounding in the first calculation.

Now to calculate the present value of an annuity due:

Use the formula

PV Annuity Due = C × [i_1 − (1 + i)−n] × (1 + i)

Plugging in the values:

= Rs. 1,000 × [0.05(1− (1 + 0.05)−5] × (1 + 0.05)

= Rs. 1,000 × 4.33 × 1.05

= Rs. 4,545.95

Example 3.Find the amount of annuity of Rs. 4,000 per annum for 10 years reckoning compound interest at 10% per annum.

Solution:

Considering immediate annuity, the required amount

= 4000 $(1 + 10/100)^9$ + 4000 $(1 + 10/100)^8$ + … + 4000 $(1 + 10/100)^1$ + 4000

For the sake of simplicity, let's write this in the reverse order. Therefore, we have

4000 + 4000 (1 + 10/100) + 4000 $(1 + 10/100)^2$ + … + 4000 $(1 + 10/100)^9$

= 4000 [1 + 1.1 + $(1.1)^2$ + … + $(1.1)^9$] … because (1 + 10/100) = 1.1

= 4000 {$(1.1)^{10}$–1/1.1–1} = 4000{(2.594–1)/0.1} = 4000×1.594/0.1 = Rs. 63,760 (Sixty three thousand even hundred and sixty).

Example 4 .Calculate the future value of the ordinary annuity and the present value of an annuity due where cash flow per period amounts to rs. 1000 and interest rate is charged at 0.05%.

Solution:

Using the formula to calculate future value of ordinary annuity = **C × [$(1 + i)^{n-1/i}$**

$= \text{Rs. } 1{,}000 \times [0.05\ (1 + 0.05)^{5-1}]$

$= \text{Rs.} 1,\ 000 \times 5.53$

$= \text{Rs. } 5{,}525.63$

Note that the one-cent difference in these outcomes, Rs. 5,525.64 vs. Rs. 5,525.63, is because of rounding in the first calculation.

Now to calculate the present value of an annuity due:

Use the formula

PV Annuity Due = $\mathbf{C \times [i_1 - (1 + i)^{-n}] \times (1 + i)}$

Q. A man requires ₹ 2, 00,000 to purchase a house after 5 years. He has an opportunity to invest the fund in an account which can earn 6% p.a. compounded quarterly. Determine the amount that must be deposited at the conclusion of each quarter in order to have the necessary sum at the end of five years.

Q. Mr. X purchases a house for ₹2, 00,000. He agrees to pay for the house in 5 equal installments p.a.. If the money is worth 5% p.a.effective, what would be size of each investment? In case X makes a down payment of ₹50, 000 what would be the size of each installment?

6.4 Understanding Loan Amortization and Annuity Payments

What Is Amortization?

Amortization is an accounting method used to gradually reduce the book value of a loan or intangible asset over a specified period. For loans, amortization involves distributing the loan payments over time. When applied to assets, amortization works similarly to depreciation.

- Amortization is the process of gradually reducing the value of a loan or intangible asset over time.
- Lenders, like banks or financial institutions, use amortization schedules to outline how a loan will be repaid by a set maturity date.
- In accordance with the matching principle in accounting standards, intangible assets are written off (expensed) over time to match their cost with the revenue they contribute to.

  • When loan payments are insufficient to pay interest, negative amortization takes place, which results in an increase in the outstanding balance rather than a decrease.

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6.4.1Amortization of Loans

A loan amortization schedule provides a detailed breakdown of each payment, showing how much of each installment goes toward interest and principal till the loan is fully paid off. Over time, more of the payment is used to lower the principal balance, but initially, a larger percentage of the payment is applied to interest.

How to Calculate Loan Amortization?

The formula to calculate the monthly principal due on an amortized loan is as follows:

Principal Payment = TMP – (OLB)

Where:

TMP = total monthly payment

OLB = 47 outstanding loan balance

When you take out a loan, the total monthly payment is usually specified. However, if you're attempting to compare or estimate monthly payments based on a particular set of parameters, such as the loan amount and interest rate, you might also need to compute the monthly payment. For any reason, the total monthly payment can be calculated using the formula below: Where, A = Payment amount

P = Initial loan amount or Principal

r = interest

n = Total number of payments

where: I =Monthly interest payment=Number of payments

Your annual interest rate must be divided by twelve. Your monthly interest rate, for instance, would be 0.25% if your annual interest rate is 3% (0.03 annual interest rate ÷ 12 months). Additionally, you will multiply the loan term's years by 12. For instance, 48 payments would be made over the course of a four-year auto loan (four years × 12 months).

The **period** is the timing of each loan payment, often represented on a monthly basis. However, each row on an amortization represents a payment so if a loan is due bi-weekly or quarterly, the period will be the same. This column helps a borrower and lender understand which payments will be broken down in what ways. This may either be shown as a payment number (i.e., Payment 1, Payment 2, etc.) or a date (i.e. 1/1/2023, 2/1/2023, etc.).

- The amount of debt owed at the start of the period is known as the beginning loan balance. This sum represents the loan's initial amount or the amount carried over from the previous month (the beginning loan balance for this month is equal to the ending loan balance from last month).
- The **interest** portion is the amount of the payment that gets applied as interest expense. This is often calculated as the outstanding loan balance multiplied by the interest rate attributable to this period's portion of the rate. For example, if a payment is owed monthly, this interest rate may be calculated as 1/12 of the interest rate multiplied by the beginning balance. Always be mindful of how a lender calculates, applies, and compounds your annual percentage rate as this impacts your schedule. As the outstanding loan balance decreases over time, less interest should be charged each period.
- ● The remaining amount of the payment is known as the principal portion. This is the total payment amount less the amount of interest expense for this period. As the outstanding loan balance decreases over time, less interest will be charged, so the value of this column should increase over time.

Pros and Cons of Loan Amortization

Amortized loans feature a level payment over their lives, which helps individuals budget their cash flows over the long term. Amortized loans are also beneficial in that there is always a principal component in each payment, so that the outstanding balance of the loan is reduced incrementally over time.

The main drawback of amortized loans is that relatively little principal is paid off in the early stages of the loan, with most of each payment going toward interest. This means that for a mortgage, for example, very little equity is being built up early on, which is unhelpful if you want to sell a home after just a few years.

Examples of loan amortization:

This type of loan is frequently used for mortgages and auto loans. Here are some examples of loans that amortize:
1. Mortgages. One of the most popular kinds of loans that amortize over time is a mortgage. A mortgage is a type of loan used to purchase a home. Both principal and

interest are paid to the lender each month by the borrower. **2. Auto loans.** Another type of loan that amortizes is a car loan. A borrower who takes out a car loan commits to paying the lender each month until the loan is repaid. The

6.5 Practical Applications of Annuities in Personal Finance

Q. Sahar takes out an auto loan for $10,000 and a payment period of 3 years, with an interest rate of 9%. She creates an amortization chart to monitor the amortization of her loan. She starts by counting how many months the amortization of her loan may take:

Solution: 3 years x 12 months in a year = 36 months

Q. She adds 36 rows to her chart. Her loan is for $10,000, and this number is her principal for the first month because this is the total amount she has to pay back. Her interest payment is 9% per year, which she converts to a monthly interest: rate:

Solution: 9% ÷ 12 months in a year = 0.0075

Q. She then calculates how much interest per month she pays for the existing principal:

Solution: $10,000 x 0.0075 = $75 monthly interest

Q. For the Principal payment column, she calculates the minimum payment she makes toward the principal every month to pay off her principal in three years. She does this by dividing the principal by the number of months in her loan period:

Solution: $10,000 ÷ 36 = $278.

Example of amortization of an intangible asset

Here's an example of the amortization of an intangible asset:

Q.TechINC. buys a patent for an invention it intends to produce and sell for $50,000. TechINC anticipates that within five years, the technology of its invention may no longer be valuable, so the patent may only be useful for that time. On TechINC's income statement, it records an asset of $50,000 for the patent. Once the usefulness of the patent is over, TechINC can't sell the patent or generate profit from it, so it has a residual value of $0.

TechINC divides the value of the patent by the years of its useful life:

$50,000 ÷ 5 = $10,000Reflecting this amortization, TechINC's income statement after one year is:

Assets
Patent: $50,000
Accumulated amortization: -$10,000
Net asset value: $40,000

6.5 Practical Examples of Annuities in Personal Finance

Suppose you receive a lump sum of Rs. 94,000 at the end of 8 years after

Question 1. Paying annuity Rs. 8,000 for 8 years. What is the implicit rate (i) in this?

First of all find $FVIFA_{in}$

96000 = 8000

$FVIFAi_8 = \ = 12$

Examine the future value annuity table. The row that corresponds to the 8-year period is 12.300, which is below the 12% column and close to 12. As a result, interest rates are lower than 12%.

Finding the Annual Annuity

Now, take an example where the total annuity future value (received or paid), rate of interest and the period is known. We must determine the annual annuity amount.

How much you should deposit in a bank annually so that you get Rs. 1, 50,000 at the end of 10 years at 10% rate of interest?

Annual Annuity = 1,50, 000

= Rs. 1,50,000

` = Rs. 9412.05

So you should deposit Rs. 9,412.05 in a bank every year for 10 years in order to get

Rs. 1, 50,000 at the end of 10 years

Q2. How much a person should save annually to accumulate Rs. 1,00,000 for his daughter's marriage by the end of 10 years, at the interest rate of 8%.

Solution:

Annual Annuity = 1, 00,000

Annual Annuity = 1, 00,000

= 1, 00,000

= Rs. 6903

A person should save Rs. 6,903 annually for 10 years to get Rs. 1,00,000

6.6 Unit summary

6.7 Check your progress

The TVM is the notion that money, such as Rs. 1 received today, has a different value than Rs. 1 received at any other time in the future. In other words, funds received now are worth more than funds received laterEither present or future values can be used to express these equivalent values. The discounting method can be used to convert the future value into the present value, and the compounding technique can be used to convert the present value into a future value.

1) Describe the "time value of money." What part does the interest rate play in it?

2) A person deposits Rs,.1000 today, Rs.2000 in two years and Rs. 5000 in five years. He withdraws Rs. 1500 in three years and Rs . 1000 in seven years. How must he will save after 8 years if interest rate is 7%? What is the present value of these cash flows?

3) If a deposit of Rs.3000 is made today and the interest received is 10% yearly, how much the deposit will grow after7 years and 11 years?

4) The goal is to have Rs. 20,000 by the end of ten years. 12% is the discount rate. How much should you have annually?

5) Assuming a 5% interest rate, calculate the present value of the following cash flows.

Year	Cash flows
1	Rs. 1000
2	Rs. 2000
3	Rs. 3000
4	Rs.4000
5	Rs. 5000

Unit 7: Introduction to mensuration

7.1 Introduction

7.2 Unit objectives

7.3 Area and Perimeter: Basic Calculations for Common Shapes

7.4 Surface Area and Volume of 3D Shapes (Cylinders, Cones, Spheres)

7.5 Applying Mensuration to Real-World Problems

7.6 Complex Shapes and Composite Figures

7.7 Unit summary

7.8 Check your progress

7.1 Introduction:

Measuring geometric shapes and figures is the focus of the fundamental mathematical field of mensuration. It encompasses the study of areas, volumes, surface areas, and perimeters of two-dimensional and three-dimensional objects. This chapter introduces key concepts and formulas used to calculate these measurements for various shapes such as squares, rectangles, triangles, circles, cylinders, cones, spheres, and more. Mensuration has practical applications in fields like architecture, engineering, and everyday problem-solving, making it an essential topic for understanding spatial relationships and quantifying physical spaces. Through this chapter, students will develop the ability to solve real-world problems by applying mathematical reasoning to geometric dimensions.

7.2 unit objectives:

By the end of this unit, students will be able to:

1. Understand and define key terms related to mensuration, such as area, perimeter, volume, and surface area.
2. Compute the surface area and volume of three-dimensional figures, such as cubes, cuboids, cylinders, cones, spheres, and hemispheres.
3. Solve real-life problems involving the measurement of areas and volumes of various objects and spaces.

7.3 Area and perimeter: basic calculation for common shapes

The two basic characteristics of two-dimensional shapes are area and perimeter. establishing the shape's dimensions and the length of its perimeter. Understanding the areas of 2D shapes makes it simple to calculate the surface areas of 3D bodies, and knowing the perimeter of any 2D closed shape allows us to determine its length.

Definition of Area

Area is a measure of a region’s size on a surface.

In simple words, area helps us to know how much space is occupied by a closed surface in a plane. The area is defined only for closed shapes. For a 2D geometric entity, area depends upon the shape and dimensions of the entity. It is calculated in sq units of units2 (eg: cm2, m2).

Area and Perimeter Formulas

Area and Perimeter Formulas for all Shapes			
Shape	**Area**	**Perimeter**	**Variables description**
Triangle	A = 1/2(b × h)	P = a + b + c	b = base, h = height a,b and c are sides of triangle
Rectangle	A = l × b	P = 2(l+b)	l = length, b = breadth
Square	A = s × s	P = 4 × s	s = side
Circle	A = πr2	P = 2πr	r = radius, π = 22/7 or 3.14
Ellipse	A = π×b	P = π(a+b)	a = semi major axis b = semi minor axis
Parallelogram	A = b × h	P = 2(a+b)	b = base, h = height a and b are the opposite sides
Rhombus	A = 1/2 (d1 × d2)	P = 4 × a	d1, d2 = diagonals a = side of rhombus
Trapezium	A = 1/2 × (a+b) × h	P = Sum of all Sides	a,b = length of parallel sides,

Area and Perimeter Formulas for all Shapes			
Shape	Area	Perimeter	Variables description
			h = height

For polygons, perimeter can be calculated as sum of lengths of its sides. And, for a regular polygon, i.e a polygon having equal sides, perimeter is calculated as n × a, where **n** is number of sides or edges of the polygon and **a** is the measure of its one side .

Area and Perimeter of All Shapes

The 2D shapes have some specific properties related to their dimensions and orientation of their dimensions which they adhere to. Every shape have defined formulae to calculate their area and perimeter.

Triangle

A triangle is closed figure having three sides. It has three vertices.

The side to which the perpendicular meets is called as the base of a triangle.

Rectangle

A rectangle is a four-sided polygon with parallel, equal opposite sides. Every angle in a rectangle is 90 degrees.
Rectangle length refers to the longer side of a rectangle, while width or breadth refers to the shorter side.

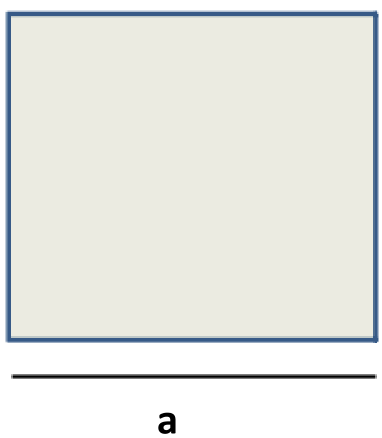
Area and Perimeter Formula of Rectangle :

Area of a rectangle = length × breadth

Perimeter of a rectangle = 2 × (length + breadth)

Square

Any four-sided polygon with equal and parallel sides is called a square. Additionally, every angle in a square has a measure of 90°. Therefore, a square can be defined as a particular kind of rectangle with equal sides on all four sides.



Area and Perimeter of Square:

Area of a Square = (side) × (side)

Perimeter of a Square = 4 × (side)

Parallelogram

A four-sided polygon with equal and parallel opposite sides is called a parallelogram. The height of a parallelogram is the distance measured perpendicularly between two opposing sides. The base of a parallelogram is the length of those sides.

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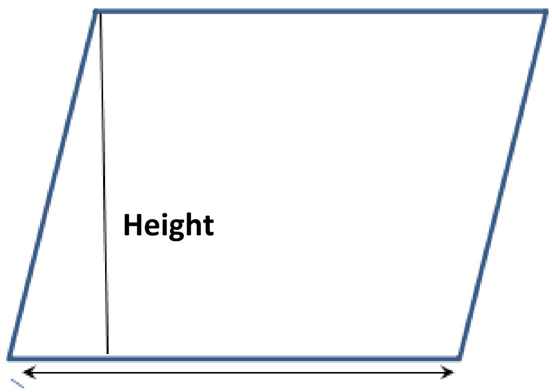
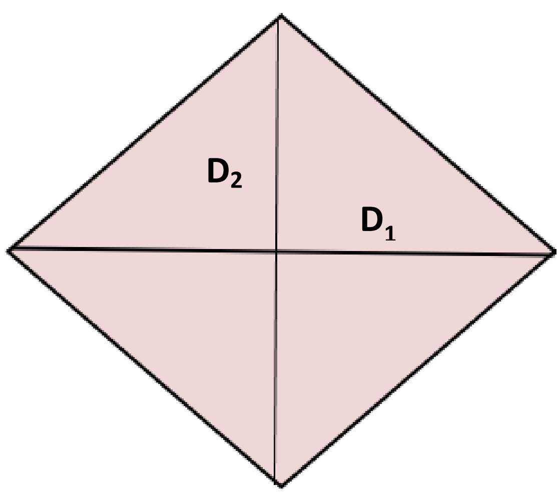


Rhombus

The length of a rhombus's diagonals is used to determine its area.



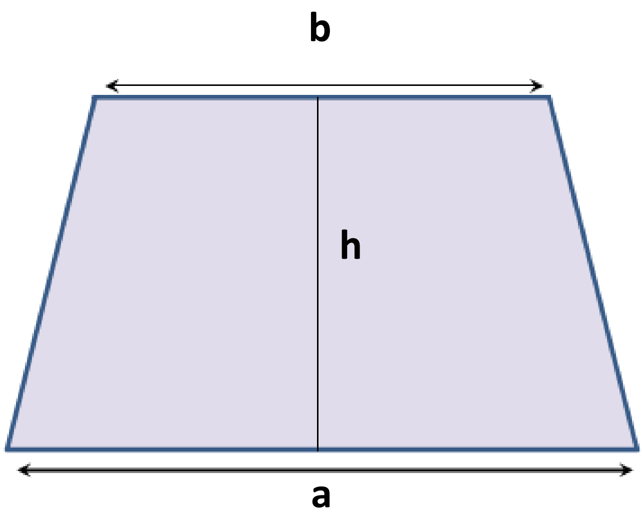
Area and Perimeter of Rhombus :

Area of a Rhombus = 1/2 × (Product of diagonals)

Perimeter of a Rhombus = 4 × side

Trapezium

The trapezium is a four-sided polygon with two opposing sides that are parallel to each other. The other two sides might or might not be parallel.



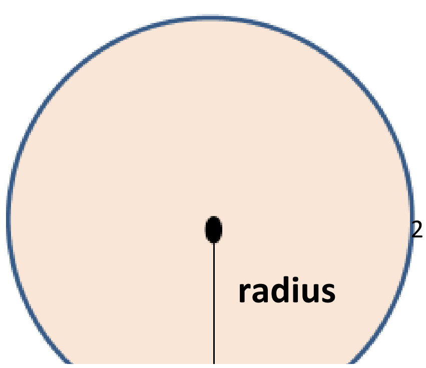
Area and Perimeter of Trapezium :

Area of a trapezium = 1/2 × (Sum of parallel sides) × (height)

Perimeter of a trapezium = (Sum of all 4 sides)

Circle

Any round shape with all points on it equally spaced from its center is called a circle. This distance is known as the circle's radius. The perimeter of a circle is known as its circumference.



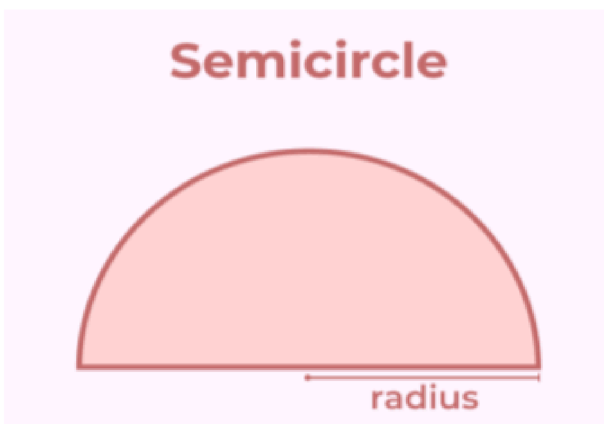
Area and Perimeter of Circle :

Area of a Circle = πr^2

Perimeter of a Circle = $2\pi r$

Semicircle

A semicircle is half of the circle whose one side is curved and other side is bounded by the diameter of the circle.



Area and Perimeter of semicircle :

Area of Semicircle = $1/2 \times \pi \times r2$

Perimeter of Semicircle = $\pi r + 2r$

Difference between Area and Perimeter

The differences between Area and Perimeter are listed in form of a table below:

Area vs Perimeter	
Area	**Perimeter**
Area is a measure of a region's size on a surface. The region is a closed 2D figure.	Perimeter is a measure of the length of boundary of any closed 2D shape.
Area is expressed in square units, such as m2, cm2, mm2, etc.	It is expressed in units, such as m, cm, mm, etc.
Example: The space occupied by a park.	Example: The length of boundary of park.

Example 1: Find the values of perimeter and area for rectangular park having length as 40 m and the breadth as 50 m.

Solution:

Given,

Length of rectangle, l = 40 m

Breadth of rectangle, b = 50 m

We know that,

Perimeter of rectangle = 2(l+b) = 2×(40+50) = 2 × 90 = 180 m.

Area of rectangle = l × b = 40 × 0 = 2000 m2

Thus,

Perimeter = 180 m. and Area = 2000 m2

Example 2: A circular running track has a radius of 7 meters. Find its circumference. Take π = 22/7.

Solution:

We have,

Radius, r = 7 m and Circumference of a Circle = $2\pi r$

Therefore,

Circumference = 2 × (22/7) × 7 = 44 meters

Thus, circumference of the circular track comes out to be 44 meters.

Example 3: The opposite sides of a parallelogram have values as 12 units and 8 units. Find the value of its perimeter.

Solution:

We know that,

Perimeter of parallelogram = 2 × (Sum of opposite sides)

Thus,

Perimeter = 2 × (12+8) = 2 × 20 = 40 units

7.4 3D Shapes' Surface Area and Volume

• Volume • Surface Area

In general, surface area can be divided into three main categories:

1. Curved Surface Area (CSA): This is the total area of a three-dimensional object that includes all of its curved areas.

2. Lateral Surface Area (LSA): This is the total area of all flat and curved surfaces, minus base areas.

3. A 3D object's Total Surface Area (TSA) is the sum of its surfaces, including its base.

The total area that a solid or three-dimensional shape takes up is referred to as its volume. The unit of measurement is cubic

7.6 problems on complex shapes and figures

Polyhedrons

- . 3D shapes are polyhedrons. As previously stated, polyhedra are solids with straight sides that have the following characteristics:
- Polyhedrons need to have flat sides, called faces, and straight edges.
- They have to have vertices, or corners.

Polyhedrons are classified as regular and irregular polyhedrons, as well as convex and concave polyhedrons, much like polygons in two-dimensional shapes.

Among the most prevalent instances of polyhedra are:

Cube: This shape has twelve edges, eight vertices, and six square faces.

Cuboid: A cuboid has twelve edges, eight vertices, and six rectangular faces.

Pyramid: This shape consists of a single vertex, flat faces, straight edges, and a polygon base.

Prism: The sides of a prism are flat parallelograms with identical polygon ends.

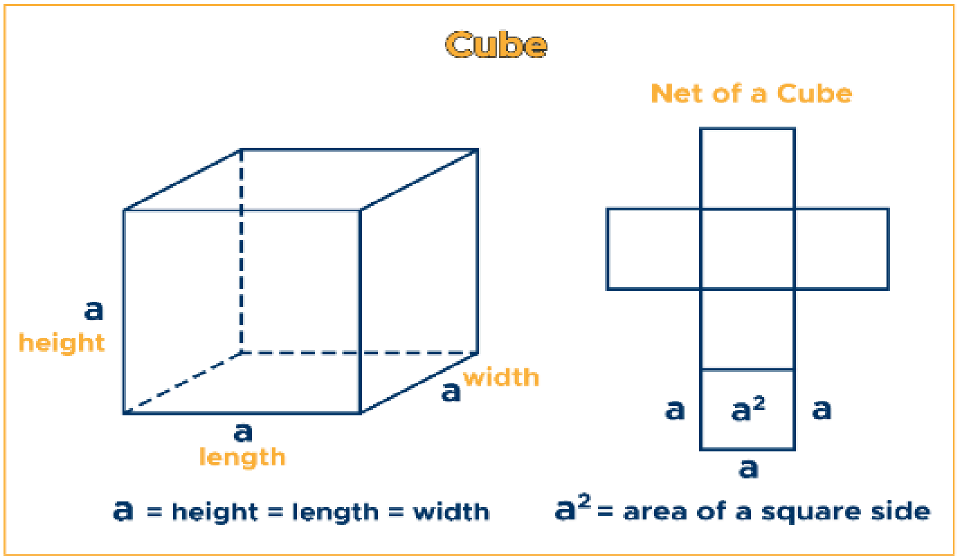
Other examples of regular polyhedrons include tetrahedrons, octahedrons, dodecahedrons, icosahedrons, and more. The faces of these regular polyhedrons, sometimes referred to as platonic solids, are identical.

For instance, a cube, a common example of a polyhedron, has 6 faces, 8 vertices, and 12 edges.

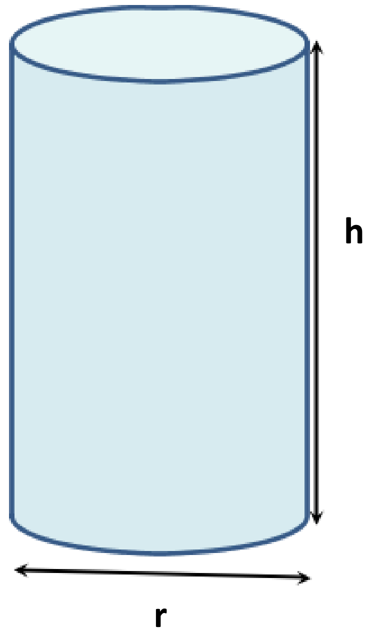
Curved Solids

Curved solids are 3D shapes with curved surfaces. Some examples of curved solids include:

- Sphere : A sphere is a round shape, with all points on the surface equidistant from the center.
- Cone : A cone has a circular base and a single vertex.
- Cylinder : A cylinder has parallel circular bases, connected by a curved surface.



Curved surface area, except its center, corresponds to the area of only the curved portion of the shape (s). It is frequently referred to as the lateral surface area for shapes like cones. "The area which includes only the curved surface area of an object or lateral surface area of an object by excluding the base area of an object" is the definition of the lateral surface area. Another name for the Lateral Surface Area is the Curved Surface Area.



Volume

The table given contains the Total Surface Area, Curved Surface Area/Lateral Surface Area, and Volume of various shapes.

Name of Shape	Curved Surface Area	Total Surface Area	Volume
Cuboid	2h(l + b)	2(lb + bh + hl)	l * b * h
Cube	4a2	6a2	a3
Cylinder	2πrh	2πr(r + h)	πr2h
Sphere	4πr2	4πr2	4/3π r3
Cone	πrl	πr(r + l)	1/3π r2h
Hemisphere	2πr2	3πr2	2/3π r3

Q. 1: Two cubes with 512 cm3 of volume each are connected end to end. Determine the resulting cuboid's surface area.

Solution:

Given,

Volume (V) of each cube is = 512 cm3

we can now imply that a3 = 512 cm3

∴ Side of the cube, i.e. a = 8 cm

The final cuboid will now have a height of 16 cm and a width and length of 8 cm each.
Thus, the cuboid's surface area (TSA) is equal to 2(lb + bh + lh).

Now, by putting the values, we get,

= 2(8 × 16 + 8 × 8 + 16 × 8) cm2

= (2 × 320) = 640 cm2

Hence, TSA of the cuboid = 640 cm^2

Q2. We have a cylindrical candle, 14 cm in diameter and of length 2cm. It is melted to form a cuboid candle of dimensions 7 cm × 11 cm×1 cm. How many Cuboidal candles can be obtained ?

Solution:

Dimensions of the cylindrical Candle:

Radius of cylindrical candle = 14/2 cm = 7 cm

Height/Thickness=2 cm

Volume of one cylindrical candle = πr2h = π x 7 x 7 x (2) cm3 = 308 cm3 .

Volume of cuboid candle = 7 x 11 x 1 = 77 cm3

Hence, number of Cuboidal candles = Volume of cuboid candle/Volume of one cylindrical candle = 308/77 = 4

Hence we can get 4 Cuboidal shaped candles

Q3. A woman wishes to construct a spherical clay toy ball with a radius that matches the radius of her current bangle. She additionally desires that the bangle's area equal the sphere's volume because it is round in shape. Determine the bangle's radius that she is wearing.

Solution:

Let r be the radius of the bangle as well as the sphere,

We have been given that the volume of the sphere is equal to the area of the bangle:

Hence,

$\pi r2 = 4/3\ \pi r3$

$\Rightarrow r = 3/4$

Hence the radius of the bangle is 3/4 units.

Q4. Find the lateral surface area of a cylinder with a base radius of 6 inches and a height of 14 inches.

Solution:

Given radius r = 6, height h = 14

LSA = 2∏rh

= 2 * ∏ * 6 * 14

= 168∏

= 527.787

= 528.

The LSA of given cylinder is 528cm.

1. Calculate the total surface area of a cylinder that is 8 cm high and has a radius of 4 cm.

2. Find the volume of a cone with radius 6 inches and height 10 inches.

3. Calculate the surface area of a rectangular prism with length 7 meters, width 4 meters, and height 6 meters.

7.7 unit summary

Mensuration is that part of mathematics that focuses on measuring the parameters of geometric shapes, including their perimeter, area, surface area, and volume. It is divided into two categories: 2D shapes and 3D shapes. For 2D shapes, the perimeter represents the total distance around the boundary, while the area measures the space enclosed by the shape. For 3D shapes, mensuration involves calculating surface area, which is the total area of all faces, and volume, which measures the space occupied by the object.

7.8 Check your Progress

Q. A square field has an area of 24200 square meters. At 6.6 km/h, how long will it take a woman to cross the field diagonally?

1. 3 minutes
2. 2 minutes
3. 2.4 minutes
4. 2 minutes 40 seconds

Q. . A cart's front and rear wheels have circumferences of thirty feet and thirty-six feet, respectively. When the front wheel has completed five more revolutions than the rear wheel, how far has the cart traveled?

1.20 ft

2.25 ft

3.750 ft

4.900 ft

Q. How many 1 cm cubes can be cut from a 5 cm cube? What is the ratio of the larger cube's surface area to the total of the smaller cubes' surface areas?

1. 1: 6

2.1: 5

3.1: 25

4.1: 125

Q Given a triangle with sides measuring 72, 75, and 21, what is the triangle's radius?

a. 37.5
b. 24
c. 9
d. 15

Q. A 4 cm cube is cut into 1 cm cubes. What is the percentage increase in the surface area after cutting?

1. 4%
2. 300%
3. 75%
4. 400%

Unit 8: Set theory and applications

8.1 Introduction

8.2 Unit objectives

8.3 Introduction to Sets: Definitions, Types, and Basic Operations

8.4 Union, Intersection, and Complement of Sets

8.5 Venn Diagrams and Applications in Problem Solving

8.6 Applications of Set Theory in Probability and Data Organization

8.7 Unit summary

8.8 Check your progress

8.1 introductions to sets: definitions, types, and the basic operations.

A subfield of logical mathematics called set theory examines collections of objects and the operations that can be performed on them.
To put it simply, a set is a group or collection of objects. For instance, a football team's players are its objects, and the team as a whole is a set.

8.2 Unit objectives:

By the end of this unit, students will be able to:

1. **Define and Understand Sets**: Understand the basic concepts of sets, including elements, notation, and types of sets (finite, infinite, equal, null, etc.).
2. **Identify Set Operations**: Perform set operations such as union, intersection, difference, and complement, and understand their properties and applications.
3. **Represent Sets**: Use various methods to represent sets, including roster form, set-builder notation, and Venn diagrams.
4. **Understand Subsets and Power Sets**: Define and identify subsets, proper subsets, and power sets of a given set and understand their significance.

8.3. Introduction to sets: Definitions, Types, and basic operations

Definition

One definition of a set is "a well-defined collection of objects." A collection is clearly defined as consisting of natural numbers, for instance, and a set of natural numbers will include all natural numbers as its members.A collection of natural numbers is expressed as follows:

N= {1,2,3,4.....}

Examples of sets

Below are a few typical instances of sets:
A collection of natural numbers N = {1, 2, 3, 4....}
An even number set: E = {2,4,6,8....}

The prime numbers are {2,3,5,7}.
Z = {...., -4,-3,-2,-1,0,1,2,...} is a number set.

Some common sets

Below are the symbols used to represent common sets in set theory.

Symbol	**Set Name**	**Description**	**Example**
N	Set of Natural Numbers	The set of all positive integers. Starting from 1.	{1, 2, 3, 4, ...}
Z	Set of Integers	The set of all positive, negative, and zero integers.	{..., -3, -2, -1, 0, 1, 2, 3, ...}
R	Set of Real Numbers	The set of all real numbers, including positive and negative rational and irrational numbers.	{..., -3.14, -2, -1, 0, 1, 2, 3.14, ...}
C	Set of	The set of all numbers that can be	{1 + 2i, 3 – 5i,

Symbol	Set Name	Description	Example
	Complex Numbers	expressed as a + bi, where a and b are real numbers and i is the imaginary unit (√(-1)).	√2, -πi, ...}
Q	Set of Rational Numbers	The set of all numbers that can be expressed as a fraction p/q, where p and q are integers and q ≠ 0.	{1/2, -3/4, 5, 0.75, ...}

terms Related to set theory

The following list includes some of the key terms associated with sets.
components of a set.
The term "elements of the set" refers to the items that make up a set. The symbol for these elements is "∈," which stands for "belongs to Important."

For instance: Since 1, 2, 3, and so on are objects in the set of natural numbers, they are also elements of the set. Another way to express that 1 is a member of set N is as 1 ∈N.
A set's cardinal number.
The Cardinal Number of a set is the total number of elements in the set.

For instance, if P is a collection of the first five prime numbers, as indicated by P = {2,3,5,7,11}, then set P's cardinal number is 5.
n(P) or |P|= 5 represents the cardinal number of a set P.

Set representation

- Roster form
- Set builder form

  Roster form

  When a set is in roster form, its elements are listed in braces `{}` and separated by commas. For instance, the formula for a set of the first five prime numbers would be P = {2, 3, 5, 7, 11}. 23 Because there are a certain number of elements in it, this represents a finite set. When a set has an infinite number of elements, the roster form indicates that the set continues indefinitely by listing a few elements followed by dots.

In roster form, for instance, the set of natural numbers is denoted by the notation N = {1, 2, 3, 4,...}, where the dots signify the infinite nature of the set

Set builder Form

In Set Builder Form, a rule or statement that outlines the shared traits of all the elements is written in place of a list of the elements enclosed in braces.

Types of sets

Sets can be classified according to a variety of criteria.

A singleton set

Example:

Set A= {1} is a singleton set as it has only one element, that is, 1.

Set P = {a : a is an even prime number} is a singleton set as it has only one element 2.

Similarly, all the 50 sets that contain only one element are known as singleton sets.

Empty Set

Null sets and void sets are other names for empty sets. These are the sets that don't contain any elements. The symbol for them is φ.

Example:

- Set A= {a: a is a number greater than 5 and less than 3}
- Set B= {p: p are the students studying in class 7 and class 8}

Finite Set :

Finite Sets are those which have a finite number of elements present, no matter how much they're increasing number, as long as they are finite in nature, They will be called a Finite set. Example:

● Set A= {a: an is the whole number less than 20}

● Set B = {a, b, c, d, e}

Infinite Set

Infinite sets are those that contain an infinite number of elements; these are the kinds of sets where it is difficult to count the number of elements.
Set A= {a: an is an odd number} is an example.

Set B is equal to {2,4,6,8,10,12,14,.....}

Equal Set

23 Equal sets are two sets with an equal number of elements and the same elements. Even if the set's components are repeated or rearranged, they will still be equal sets.
For instance:
Set A is equal to {1, 2, 6, 5}.

Set B is equal to {2, 1, 5, 6}.
The elements in the example above are 1, 2, 5, and 6. Consequently, A = B

Equivalent Set

Sets that contain the same number of elements are called equivalent sets. It's crucial to remember that while the elements in the two sets might differ, there are exactly the same number of them. For example, two sets are equivalent if one contains six elements and the other likewise contains six.
For instance:
{2, 3, 5, 7, 11} is the set A.

{p, q, r, s, t} = Set B
Since both Set A and Set B contain five elements, they are equivalent sets.

Subset

Set A will be called the Subset of Set B if all the elements present in Set A already belong to Set B. The symbol used for the subset is ⊆

If A is a Subset of B, It will be written as A ⊆ B

Example:

Set A= {33, 66, 99}

Set B = {22, 11, 33, 99, 66}

Then, Set A ⊆ Set B

Power Set

Any set A's power set is the set that includes every subset of that set. It can be read as Power set of A and is represented by the symbol P(A).

For instance: What is the power set of any set A = {a,b,c}?

Solution:
Power Set P(A) is,
P(A) = {φ, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}}

Universal Set

A set that includes every element of the other sets is called a universal set. One could argue that every set is a subset of the universal sets. U stands for the universal set.
For instance: Find the universal set that contains both Sets A = {a, b, c, d} and B = {1,2}.
Answer:

The formula for the Universal Set U is U = {a, b, c, d, e, 1, 2}.

Disjoint Sets

Disjoint sets are any two sets, A and B, that do not share any elements. Since φ is the intersection of the disjoint set, A∩B = φ for sets A and B.

Example: Check whether Set A ={a, b, c, d} and Set B= {1,2} are disjoint or not.

Solution:

Set A ={a, b, c, d}
Set B= {1,2}

Here, A∩B = φ

Thus, Set A and Set B are disjoint sets.

Summarizing Types of Set

There are different types of sets categorized on various parameters. Some types of sets are mentioned below:

Set Name	Description	Example
Empty set	A set containing no elements whatsoever.	{}
Singleton set	A set containing exactly one element.	{1}
Finite set	A set with a limited, countable number of elements.	{apple, banana, orange}
Infinite set	A set with an uncountable number of elements.	{natural numbers (1, 2, 3, ...)}
Equivalent Sets	Sets that have the same number of elements and their elements can be paired one-to-one.	Set A = {1, 2, 3} and Set B = {a, b, c} (assuming a corresponds to 1, b to 2, and c to 3)
Equal set	Sets that contain exactly the same elements.	Set A = {1, 2} and Set B = {1, 2}
Universal set	A set containing all elements relevant to a specific discussion.	The set of all students in a school (when discussing student grades)
Unequal Sets	Sets that do not have all the same elements.	Set A = {1, 2, 3} and Set B = {a, b}
Power set	The set contains all possible	Power Set of {a, b} = { {}, {a},

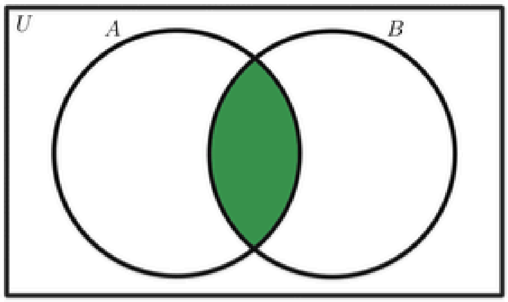
Set Name	Description	Example
	subsets of a given set.	{b}, {a, b} }
Overlapping Sets	Sets that share at least one common element.	Set A = {1, 2, 3} and Set B = {2, 4, 5}
Disjoint set	Sets that have no elements in common.	Set A = {1, 2, 3} and Set B = {a, b, c}
Subset	A set where all elements are also members of another set.	{1, 2} is a subset of {1, 2, 3}

8.2 Basic operations on set:

- Union of Sets
- Intersection of Sets
- Difference of Sets
- Complement of Sets

Intersection of Sets

The formula $A \cap B = \{x : x \in A \text{ and } x \in B\}$



$A \cap B$

Example:

If A = {2, 3, 5, 7} and B = {1, 2, 3, 4, 5}

then the intersection of set A and B is the set A ∩ B = {2, 3, 5}

In this example 2, 3, and 5 are the only elements that belong to both sets A and B.

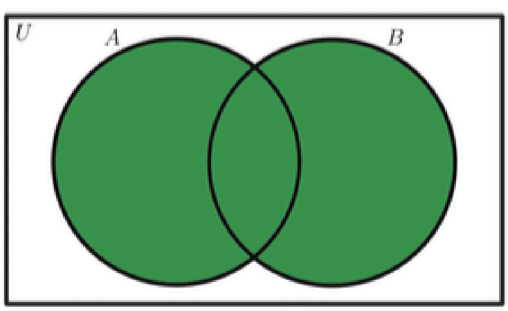
8.4 Union intersection and complement of sets

Union of Sets

Combining two sets All of the elements that are in either A or B, or both, are included in the set A and B. In formal terms, it is written as

A∪B={x: x ∈ A or x ∈ B}

In the following figure, the shaded area is the union of sets A and B



$A \cup B$

Example:

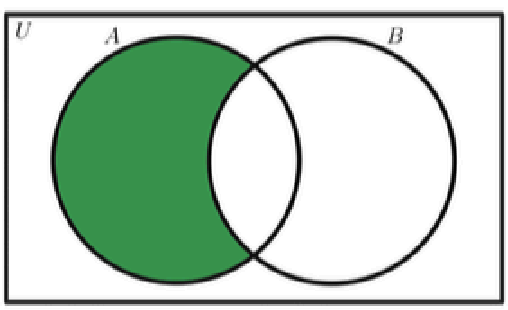
If A = {2, 4, 8} and B = {2, 6, 8}
then the union of A and B is the set A ∪ B = {2, 4, 6, 8}

In this example, 2, 4, 6, and 8 are the elements that are found in set A or in set B or in both sets A and B.

Complement or Difference between Sets

The set that contains all of the elements in A but not in B is known as the relative complement or set difference of two sets, A and B. Formally, this is stated as follows: Note: $A - B = \{x: x \in A \text{ and } x \notin B\}$

This can also occasionally be written as A \ B. The difference set between sets A and B is represented by the shaded area in the following image.



$A - B$

Note: A – B is equivalent to A ∩ B’ i.e., $A - B = A \cap B'$

Example:

If A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {2, 3, 5, 7}
then A – B = {1, 4, 6, 8, 9, 10}
and further B – A = ∅

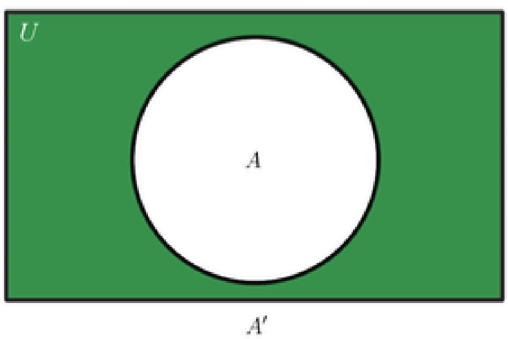
Universal set and Absolute complement

Universal set

A universal set is the collection of all the items under consideration at the time. The capital letter U is typically used to represent it. For instance, the set of alphabets might be the universal set for a set of vowels.

It should be noted that every set is a subset of the universal set.

Absolute complement



The absolute complement is sometimes just called complement.

Note: A’ is equivalent to U – A i.e., A’ = U – A

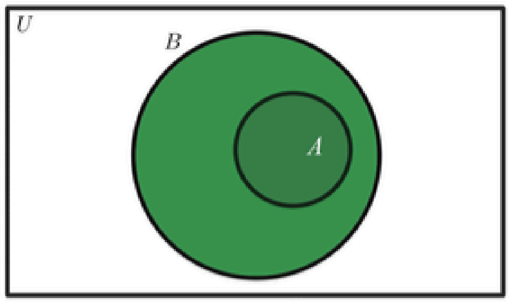
If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = {1, 2, 3}
then A’ = {4, 5, 6, 7, 8, 9, 10} = U – A

$A \subseteq B$, if $\forall$ x{x $\in$A=>x $\in$ B}

Subset and Proper Subset

Subset

If every element in two sets, A and B, is also in B, then A is a subset of B. A and B may be equivalent. Formally, this is written as



In the following image, set A is a subset of B

Example:

1. An empty set **(or null set) is a subset of every set.**

A ⊂ B, $if \forall x${x ∈A=> x∈B} and A $\neq B$

Example:

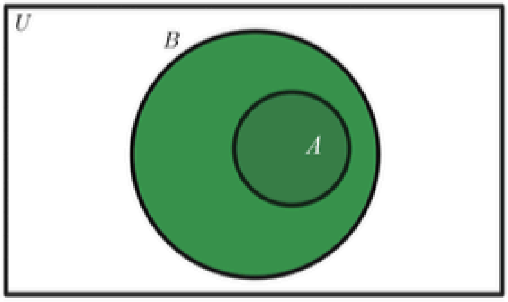
Ø is a subset of the set {1, 2, 3, 4}

Example:

For the set C = {1, 2, 3}, there are 23 = 8 possible subsets
they are Ø, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}

Superset and Proper Superset

Superset



$B \supseteq A$

In the following image, set B is the superset of set A

$B \supset A$, if $A \subseteq B$ and $A \neq B$

Examples:

- If A = {2, 4} and B = {1, 2, 3, 4, 5, 6, 7, 8}
  then B is the superset of A, because A is a subset of B

- If A = {11, 12} and B = {11, 12 } then B is the super set of A

Proper superset (also called strict superset)

- If A = {1, 2, 3} and B = {0, 1, 2, 3, 4, 5}
  then B is a proper superset of A, because A is a subset of B and A ≠ B

- If A = {2, 4, 6} and B = {2, 4, 6} then B is **not** a proper superset of A, because A = B

8.6 Applications on set theory in Probability and Data organization

Question 1: Out of a class of 100 students, 45 indicated that they enjoyed apples, and 30 said they enjoyed both apples and oranges. Each pupil must select a minimum of one of the two fruits. Find how many students like oranges**.**

Solution:

Let U = set of all students in the class
A = set of students that like apples
B = set of students that like oranges

Given:
$|A| = 45$
$|A \cap B| = 30$
$|U| = |A \cup B| = 100$ (because every student has to choose)

We need to find how many like oranges. i.e., $|B|$

Subtract $|A| - |A \cap B|$ from both sides in (i) to get
$|A \cup B| - (|A| - |A \cap B|) = |B|$
or $|B| = |A \cup B| - (|A| - |A \cap B|)$

Substitute the given values and simplify,
$|B| = |A \cup B| - (|A| - |A \cap B|)$
$= 100 - (45 - 30)$
$= 85$

Thus the number of students that like oranges is 85

Question 2: A class consists of 120 pupils in total. Ten of them study both science and mathematics, 40 study science, and 70 study mathematics. Determine the proportion of students who

i) study math but not science.
ii) Learn science instead of math.

iii) Learn science or math.

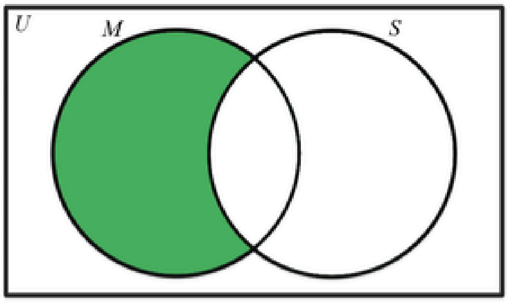
Let,
U = set of all students in the class
M = set of students that study mathematics
S = set of students that study science

Our universal set here has 120 student i.e, $|U| = 120$

Given,
$|M| = 70$

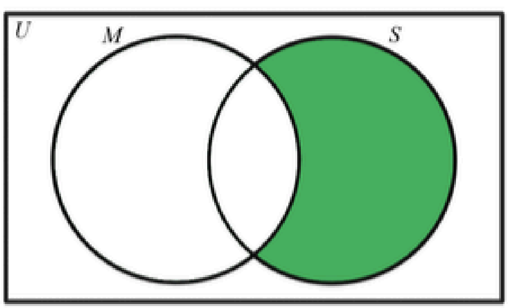
|S| = 40
|M ∩ S| = 10 (number of students that study both mathematics and science)

(i) Determining the proportion of pupils who pursue mathematics but not science. In the following image, the shaded area represents the set of students that study mathematics but not science.



We are required to find |M – S|
By the Venn diagram, we can see that |M – S| can be written as |M| – |M ∩ S|
thus,
|M – S| = |M| – |M ∩ S|
= 70 – 10
= 60

Thus the number of students who study mathematics but not science is 60



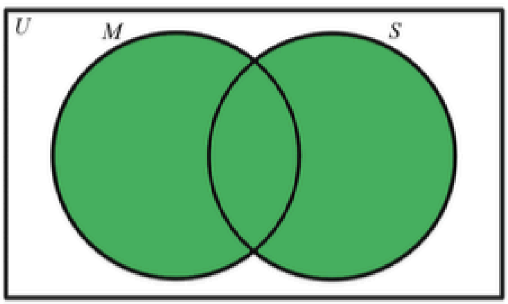
By the Venn diagram, we can see that $|S - M|$ can be written as $|S| - |M \cap S|$

thus,

$$|S - M| = |S| - |M \cap S|$$
$$= 40 - 10$$
$$= 30$$

Thus the number of students who study science but not mathematics is 30

iii) Finding the number of students who study mathematics or science. In the following image, the shaded area represents the set of students that study mathematics or science.



We are required to find $|M \cup S|$

By using the formula, $|M \cup S| = |M| + |S| - |M \cap S|$

$$|M \cup S| = |M| + |S| - |M \cap S|$$
$$= 70 + 40 - 10$$
$$= 100$$

Operation on sets-solved Examples

A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6, 7}, and C = {4, 5, 6, 7, 8}. This is the first problem. Since △ denotes a symmetric difference, find (A △ B) ∩ (B △ C).

Solution:

First, let's find $A \triangle B$:

$A \triangle B = (A \cup B) - (A \cap B)$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cap B = \{3, 4, 5\}$

$A \triangle B = \{1, 2, 6, 7\}$

Now, let's find $B \triangle C$:

$B \triangle C = (B \cup C) – (B \cap C)$

$B \cup C = \{3, 4, 5, 6, 7, 8\}$

$B \cap C = \{4, 5, 6, 7\}$

$B \triangle C = \{3, 8\}$

Finally, we find the intersection of these results:

$(A \triangle B) \cap (B \triangle C) = \{1, 2, 6, 7\} \cap \{3, 8\} = \emptyset$ (empty set)

Second problem: $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 2, 3, 4, 5\}$, and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
Make sure $(A \cup B)' = A' \cap B'$, where "" stands for complement with regard to U.

Solution:

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Find $(A \cup B)'$:

$(A \cup B)' = \{10\}$

Find A':

$A' = \{2, 4, 6, 8, 10\}$

Find B':

$B' = \{1, 3, 5, 7, 9, 10\}$

Find $A' \cap B'$:

$A' \cap B' = \{10\}$

Verify that $(A \cup B)' = A' \cap B'$:

Both sets equal {10}, so the equality holds.

8.5 Venn Diagrams and applications in problem solving.

What are Venn Diagrams?

When there are multiple groups of elements that overlap, we can say that the overlapping groups have similar elements in them. The Venn diagram is a useful tool for visually representing groups of elements of the same category.

Applications of Venn Diagrams

Venn diagrams are used in both theoretical and practical contexts, including:

- Logical Representation
- Solving Complex Mathematical Problems
- Information Visualization
- Management and Marketing
- Computer Science

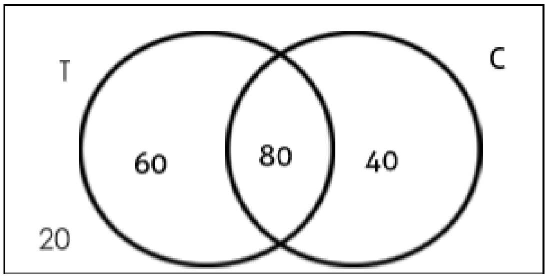
Solved examples:

Example 1: In a college, 200 students are randomly selected. 140 like tea, 120 like coffee and 80 like both tea and coffee.

- **How many students like only tea?**
- **How many students like only coffee?**

  How many students like neither tea nor coffee?
- **How many students like only one of tea or coffee?**

Solution: The given information may be represented by the following Venn diagram, where T = tea and C = coffee.



There are 60 students who prefer tea exclusively, 40 who prefer coffee exclusively, and 20 who prefer neither tea nor coffee.

● There are 60 + 40 = 100 students who prefer only one type of tea or coffee.
● n (only tea) + n (only coffee) + n (both tea & coffee) = 60 + 40 + 80 = 180 is the number of students who enjoy at least one of these beverages.

8.6 Applications on set theory

First question: Of the 40 students in the class, 22 play hockey, 26 play basketball, and 14 play both sports. What percentage of students don't participate in either game?

Solution:

Assume that students A and H are playing basketball and hockey, respectively.

$n(H) = 14$, $n(H \cap B) = 14$, and $n(H) = 22$.

The formula for $n(H \cup B)$ is $n(H) + n(B) - n(H \cap B) = 22 + 26 - 14 = 34$.

Students not playing either = Total students - $n(H \cup B) = 40 - 34 = 6$.

Solution: The set of elements that are in either A or B, or both, is $A \cup B$ (union).

$A \wedge B = [1, 2, 3, 4, 5, 7, 9\}]$.

The intersection, or $A \cap B$, is the collection of elements that are in both A and B.

$A \cap B = \{1, 3, 5\}$.

Question 3: In a survey of 60 people, 25 liked tea, 30 liked coffee, and 10 liked both. How many people liked only tea?

Solution:

Number of people who liked only tea = Number who liked tea - Number who liked both.
= 25 - 10 = 15 people liked only tea.

Question 5: Given sets X = {a, b, c, d} and Y = {b, d, e, f}, find the symmetric difference of X and Y (denoted as X Δ Y).

Solution:

X Δ Y is the set of elements in either X or Y, but not in their intersection.
Intersection X ∩ Y = {b, d}.
X Δ Y = (X ∪ Y) - (X ∩ Y) = {a, b, c, d, e, f} - {b, d} = {a, c, e, f}.

Question 6: If set C = {2, 4, 6, 8} and set D = {6, 8, 10, 12}, what are the sets C ∩ D and C ∪ D?

Solution:

C ∩ D (intersection) is the set of elements common to both C and D.

C ∩ D = {6, 8}.

C ∪ D (union) is the set of all elements in C, or D, or both.

C ∪ D = {2, 4, 6, 8, 10, 12}.

8.7 Unit Summary

Set Theory is a fundamental branch of mathematics that deals with the study of sets, which are collections of objects or elements. This unit explores the basic concepts of sets, including their definitions, types, and operations. Students learn how to represent sets using different notations like roster form and set-builder notation, and how to visualize set relationships through Venn diagrams. Key operations such as union, intersection, complement, and difference are introduced, along with the laws governing these operations. The unit also covers subsets, power sets, and the concept of a universal set, helping students understand the structure and hierarchy of sets. Furthermore, the unit delves into relations and functions, exploring how elements from one set can be related to elements of another.

Unit 9: Permutation and combination

9.1 Introduction:

The method of selecting things is called a combination and that of arranging the selected things is called permutations. Permutation and combination provide the rules of counting the different numbers in a wide variety of problems relating to statistics and quantitative techniques.

9.2 Unit objectives

By the end of this unit, students will be able to:

1. Grasp the Concepts of Permutation and Combination
2. Apply Permutation Formula: Use the permutation formula to calculate the number of possible arrangements of a set of objects, both with and without repetition.
3. Solve Problems Involving Permutations: Solve problems related to the arrangement of objects in linear and circular permutations, including those with restrictions.

9.3 Basics of permutation and combination, definition and calculations

A permutation is an arrangement is a definite order of a number of objects taken some or all at a time. It refers to the maximum possible number of arrangements that can be made of a given number of things taking one or more of them at a time in different possible orders. For example, if there are three things say a, b and c, they can be arranged in the following different possible ways taking one, two or three at a time respectively.

Possible arrangements of a, b, c in different order

Nature of pairs arrangements	possible arrangements	Total number of
Taking one at a time	a,b,c	3
Taking two at a time	ab, ac, bc, ba, ca, cb	6
Taking all at a time	abc, acb, bca, bac, cab, cba	6

9.3 Formula of Permutation and combination

A permutation refers to the selection of r items from a set of n items without replacement, where the order of the items is important.

^nP_r = (n!) / (n-r)!

Where n = number of different things given and n! is read as "n factorial" which implies the product of n (n –1) (n – 2) ! etc. r = number of tings taken at a time viz one, two, three etc.

Combination Formula

A combination is the choice of r things from a set of n things without replacement and where order does not matter.

$$_nC_r = \binom{n}{r} = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Solved Examples of Permutation and Combinations

Example 1:

Find the number of permutations and combinations if n = 12 and r = 2.

Solution:

Given,

n = 12
r = 2

Using the formula given above:

Permutation:

${}^{n}P_{r}$ = (n!) / (n-r)! =(12!) / (12-2)! = 12! / 10! = (12 x 11 x 10!)/ 10! = 132

Combination:

$nC_r = \frac{n!}{r!(n-r!)}$

$= \frac{12!}{2!(12-2!)} = \frac{12!}{2!(10!)} = \frac{12\times11\times10!}{10!} = 66$

Example 2:

In a dictionary, if all permutations of the letters of the word AGAIN are arranged in an order. What is the 49th word?

Solution:

Start with the letter A	The arranging the other 4 letters: G, A, I, N = 4! = 24	First 24 words
Start with the letter G	arrange A, A, I and N in different ways: 4!/2! = 12	Next 12 words
Start with the letter I	arrange A, A, G and N in different ways: 4!/2! = 12	Next 12 words

This accounts up to the 48th word. The 49th word is “NAAGI”.

9.4 Applications on Permutation and Combination.

Question 1: In how many ways can the letters be arranged so that all the vowels come together? Word is “IMPOSSIBLE.”

Question 2: In how many ways of 4 girls and 7 boys, can be chosen out of 10 girls and 12 boys to make the team?

Question 3: How many words can be formed by 3 vowels and 6 consonants taken from 5 vowels and 10 consonants?

Solved examples

Question 1:

Evaluate the following

(i) 6 ! (ii) 5 ! – 2 !

Solution:

(i) 6! = 1 × 2 × 3 × 4 × 5 × 6 = 720

(ii) 5! = 1 × 2 × 3 × 4 x 5 = 120

As 2! = 1 × 2 = 2

Therefore, 5 ! – 2 ! = 120-2 = 118.

Question 3: From a team of 6 students, in how many different ways can a captain and a vice-captain be selected, assuming one person cannot hold both positions?

Solution:

From a team of 6 students, how many ways can two students be selected such that each student holds only one position?Here, the no. of ways of choosing a captain and vice-captain is the permutation of 6 different things taken 2 at a time.

So, $^6P_2$ = 6! / (6 -2)! = 6! / 4! = 30

Question 4: How many distinct words, meaningful or not, can be formed using all the letters of the word "EQUATION," with each letter used exactly once?

Solution:

The no. of vowels letters in word EQUATION` = 8

n = 8

If all letters of the word used at a time r = 8

Different numbers formed = nPr

= $^{8}P_{8}$

= 8!/(8 8)!

= 8!/0!

= 8!/1

= 8!

= 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1

= 40320

Question 5:

Find the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Solution:

Take a deck of 52 cards,

To get exactly one king, 5-card combinations have to be made. It should be made in such a way that in each selection of 5 cards, or in a deck of 52 cards, there will be 4 kings.

To select 1 king out of 4 kings = $^{4}c_{1}$

To select 4 cards out of the remaining 48 cards = $^{48}c_{4}$

To get the needed number of 5 card combination = $^{4}c_{1}$ x $^{48}c_{4}$

= 4x2x 47x 46×45

= 778320 ways.

Check your progress

1. How many numbers between 99 and 1000 have at least one digit equal to 7? **[Answer: 252]**

2. A boy has 3 library tickets and 8 books of interest in the library. Out of these 8 books, he will not borrow Mathematics Part II unless he also borrows Mathematics Part I. In what ways can he select three books to borrow? **[Answer: 41]**

3. Compute n!/ r!(n-r)!, when n=5 and r=2. **[Answer: 10]**

4. How many different automobile license plates can be created if each plate consists of two distinct letters followed by three distinct digits? **[Answer: 468000]**

9.3 Applications of permutation in arrangements and sequences

A mathematical method for sorting items from the total number of items or people. In permutation, the order of objects or the way they are arranged is of high importance. We can denote permutation by the formula nPr. Where r is the number of selected objects and n is the total number of objects. Let's take an example to get a clear picture of permutation.

Q. How many four-digit numbers can be created with the numbers 2, 3, 5, 6, and each digit used only once?

We will get different numbers by arranging the given digits in different ways – 2345, 6543

So, total four digits number = 5P4

n=5,r=4

=5!/(5-4)! = 5!/1! = 5*4*3*2*1 = 120.

Classification of permutation

Permutation can be classified into three categories-

1. Permutation of n different objects when repetition is not allowed.

2. Permutation of n different objects when repetition is allowed.

Examples of permutation

Type 1 is predicated on the basic idea of counting, or more simply, on basic inquiries about the components of object selection between two given objects in the absence of any conditions.

1. How many ways are there to take the top three spots in a race with nine students?Sol- As per the question, 3 students can take the first 3 seats. This suggests that it is the question of permutation.

The first 3 places can be taken as-

$nPr = 9!/(9-3)! = 9!/6! = 9*8*7= 504$ Ways

Type2- It is based on arrangement of objects taking more than one, together or separate according to condition

1.If $2\ nP3 = n+1P3$

$nPr = n!/(n-r)!$

$2.n!/(n-3)! = (n+1)!/(n+1-3)$

$2.(n)!/(n-3)! = (n+1)(n)!/(n-2)(n-3)$

$2(n-2)=n+1$

$2n-4=n+1$

$2n-4=n+1$

$n=5.$

Conclusion

In mathematics, a permutation of a set is a loose grouping of its members into a sequence or linear order, or a rearrangement of its elements if the set is already sorted. The act or process of shifting the linear order of an ordered set is also known as “permutation.” The formula of permutation when, repetition is not allowed – $nPr=n!(n-r)!$

The formula of permutation when, repetition are allowed – $nPr= nr$

Some questions based on permutation

we can make one, two, three, four, five, and six-digit numbers.

1 digit number = 6P1 = 6!/5! = 6.

2 digit number = 6P2 = 6!/4! = 6*5=30.

1 digit number = 6P3 = 6!/3! = 6*5*4=120.

1 digit number = 6P4 = 6!/2! = 6*5*4*3=360.

1 digit number = 6P5 = 6!/1! = 6*5*4*3*2*1=720.

1 digit number = 6P6= 6!/0! = 6*5*4*3*2*1=720.

Total number of ways= 6+30+120+360+720+720=1956.

Q2. . How many different ways are there to arrange the word "DETAIL" so that the vowels only appear in odd positions?

Solution- total number of letters in word 'DETAIL' = 6

Vowels in a letter = 3

Consonants in a letter = 3

The number of possible vowel arrangements is 3P3 = 3! = 3*2=6.
3P3 = 3! = 3*2=6 is the number of possible arrangements for three consonants.

Total number of ways = (6*6)=36.

Some questions for practice

Q1. Determine how many distinct words can be created using the letters of the word "PRACTICE" without any vowels joining.

Q2. There are five people named P1,P2,P3,P4,P5. Out of 5 persons three persons are to be arranged in a line such that in each arrangement P1 must occur whereas P2 and p53 do not occur. Find the number of such possible arrangements.

9.5 understanding combination concept and calculations

What is the Combination? Combinations are the different ways that items can be chosen from a given set. To create the subsets, it is typically done without replacement. Combinations are a way to find out the total outcomes of an event where the order of the outcomes does not matter. Thus the Combination is the different selections of a given number of objects taken some or all at a time. For example, if we have two alphabets A and B, then there is only one way to select two items, we select both of them. **Combinations Formula?**

The number of possible groups of r objects each that can be created from the available n different objects can be quickly determined using the combinations formula. The factorial of n divided by the product of the factorial of r and the factorial of the difference between n and r, respectively, is the formula for combinations.

Combinations Formula:

Another name for the combinations formula is the ncr formula. We must understand what a factorial is in order to apply the combinations formula, and we have n!
nPr = (n!) / (n-r)!
n! = 1 × 2 × 3 × .... (n - 1) × n.

Combinations as Selections

Let's say we count how many three-letter combinations we can make out of those six letters. Six P33 would be that number. Think about the combinations of the letters A, B, and C. ABC, ACB, BAC, BCA, CAB, and CBA are the three ways that add up to six. Now, what we want is the number of combinations and not the number of arrangements. In other words, the 6 permutations listed above would correspond to a single combination. Differently put, the order of things is not important; only the group/combination matters now in our selection.

How To Apply Combinations Formula?

We calculate combinations using the combinations formula, and by using factorials and in terms of permutations. In general, suppose we have n things available to us, and we want to find the number of ways in which we can select r things out of these n things. We first find the number of all the permutations of these n things taken r at a time. That number would be Now, in this list of nP_r permutations, each combination will be counted r! times since r things can be permuted amongst themselves in r! ways. Thus, the total number of permutations and combinations of these n things, taken r at a time, denoted by nC_r will be:

$$nC_r = \frac{nPr}{r} = \frac{n!}{r!(n-r)!}$$

Relation between Combinations Formula and Permutations Formula

Theorem: nPr = nCr × r! (Permutations formula = Combinations formula × r!)

Proof:

Consider,

RHS, nCr × r!

= [n!/ r!(n-r)!]r!

= n!/(n-r)! = nPr

Hence, the theorem is verified.

What is Factorial?

Continued product of first n natural numbers (i.e., the product of 1, 2, 3, …, n) is denoted by symbol n! and read as factorial n.

For example, 5! = 1.2.3.4.5 = 120

In general,

n! = 1.2.3.4…..(n – 1).n

Note:

1. We define 1! = 1 and 0! = 1
2. n! is not defined when n is a negative integer or a fraction

Remarks:

- We have nCr = n!/r!(n-r)! In particular, if r = n, then nCn = n! /n! = 1
- nC0 = n! /0! (n-0)! = n!/0!n! = 1/0! = 1. Thus the formula nCr = n!/r!(n-r)! is applicable for r = 0 also. Hence, nCr = n!/r!(n-r)! , 0 ≤ r ≤ n
- nCr = n! /r!(n-r)! = n(n-1)(n-2)……..(n-r+-1)(n-r)(n-r-1)…….3.2.1 / r! [(n-r)(n-r-1)…..3.2.1]. Therefore, nCr = n(n-1)(n-2)…….r factors/ r!
- nCn-r = n!/ (n-r)![n-(n-r)]! = n!/ (n-r)! r! = nCr. Hence, nCr = nCn-r i.e., selecting r objects is same as rejecting (n-r) objects

Question 1: Evaluate 4! – 3!
Solution:
4! – 3!
= (4 × 3 × 2 × 1) – (3 × 2 × 1)
= 24 – 6

=18

Question 2: From a class of 30 students, 4 are to be chosen for the competition.
Solution:

Total students = n = 30

Number of students to be chosen = r = 4

Hence, Total number of ways 4 students out of 30 can be chosen is,

30C4

= 30! / (4!(30-4)!)

= 30! / (4!26!)

= 27,405 ways

Question 3: Nitin has 5 friends. In how many ways can he invite one or more of them to his party.

Solution:

Nitin may invite (i) one of them (ii) two of them (iii) three of them (iv) four of them (v) all of them

with combinations formula we can calculate that this can be done in 5C1, 5C2, 5C3, 5C4, 5C5 ways

Therefore, The total number of ways

= 5C1 + 5C2 + 5C3 + 5C4 + 5C5

= 5!/ (1! 4!) + 5!/ (2! 3!) + 5!/ (4! 1!) + 5!/ (5! 0!)

= 5 + 10 + 10 + 5 +1

= 31 ways

Question 4: Find the number of diagonals that can be drawn by joining the angular points of an octagon.

Solution:

A diagonal is made by joining any two angular points.

There are 8 vertices or angular points in an octagon

Therefore, by using combinations formula the Number of straight lines formed

= 8C2 = 8!/ (2! 6!)

= 8 ×7 / (2 × 1)

= 56/ 2

= 28

Which also includes the 8 sides of the octagon

Therefore, Number of diagonal

= 28 – sides of octagon

= 28 – 8

= 20 diagonals

Question 5: Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls, and 7 blue balls, if each selection consists of 3 balls of each color.

Solution:

Number of Red balls = 6

Number of white balls = 5

Number of Blue balls = 7

Total number of balls to be selected = 9

Hence, the required number of ways of selecting 9 balls from 6 red, 5 white, 7 blue balls consisting of 3 balls of each color can be calculated by using combinations formula :

= 6C3 × 5C3 × 7C3

= 6!/ (3! 3!) × 5!/ (3! 2!) × 7!/ (3! 4!)

= 20 × 10 × 35

= 7000 ways

9.6 Real world applications of permutation and combination

Here's some applications of permutations and combinations are added below as:

In Passwords and Security

Permutations and combinations are frequently used in cybersecurity and encryption to generate secure systems and strong passwords. You choose characters from a list of options, including letters, numbers, and symbols, when creating a password. There are more possible password combinations the more options you have for each character position. By doing this, hackers are less likely to use brute force attacks, which involve trying every possible combination until they find the one that works, to guess or crack your password. Therefore, we can make our passwords and systems more secure by expanding the number of possible combinations.

In Lottery and Gambling

Permutations and combinations are important in lottery games and other gambling activities. In the lottery, winning is just a permutation of number picked from the pool of numbers. It is important to get the idea of the probabilities of each possible combination because it can influence the decisions players make, however luck is also an inseparable element.

In Seating Arrangements and Event Planning

A seating plan is a very important element of the planning process regardless of whether you are organizing a theater performance, a wedding reception, or a conference. Every single individual who is attending the event is directly impacted by how well the seating system works. The number of possibilities created by permutations and combinations makes it easier for the planners to bring each guest to that exact spot where they belong. The number of possible configurations increases geometrically with the number of participants, highlighting the need for well-planned arrangements. In-depth planning of the seating arrangement would amend the whole event, leading to conversation, comfort, and satisfaction among those participating.

In Logistics and Operations Research

In logistics and operations research, it's really important to be efficient. The key is to figure out the best way to use resources to make things work better and cost less. Whether it's planning delivery routes for trucks, scheduling work shifts, or managing a factory, using different combinations of resources can help a lot. This can save time and money for companies.

9.7 Unit Summary

Permutation and combination are fundamental concepts in combinatorics, a branch of mathematics that deals with counting and arranging objects. The arrangement of items in a particular order is referred to as permutation. It is used when the order of selection matters. The formula for permutations is $P(n, r) = n! / (n - r)!$, where n is the total number of objects, and r is the number of objects to be arranged. On the other hand, Combination refers to the selection of objects without considering the order. It is used when the arrangement does not matter. The formula for combinations is $C(n, r) = n! / r!(n - r)!$, where n is the total number of objects, and r is the number of objects chosen. Understanding permutations and combinations helps in solving problems

9.8 check your progress

Q1. The number of ways in which 8 students can be seated in a line is

- 5040
- 50400
- 40230
- 40320

Q2.If ${}^{n}P_5 = 60\,{}^{n-1}P_3$, **the value of n is**

a. 6
b. 10
c. 12
d. 16

Q3. The number of squares that can be formed on a chessboard is

a. 64
b. 160
c. 204
d. 224

Q4.The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is

a. 12
b. 104
c. 144
d. 256

Q5.The number of ways 10 digit numbers can be written using the digits 1 and 2 is

a. 2^{10}
b. ${}^{10}C_2$
c. $10!$
d. ${}^{10}C_1 + {}^{9}C_2$

Q6.A coin is tossed n times, the number of all the possible outcomes is

a. 2n
b. 2^n
c. C(n, 2)
d. P(n, 2)

Unit 10 Matrix Algebra

10.1 Introduction to matrices

A matrix of order m by n, or m × n matrix, is a rectangular array of m × n real or complex numbers represented by m horizontal lines (referred to as rows) and n vertical lines (referred to as columns). [ ] or ( ) encloses such an array. We will discover the definition of matrices, their types, key formulas, and more in this notes.

10.2 Unit objectives:

Unit Objectives: Matrix Algebra

By the end of this unit on Matrix Algebra, students will be able to:

1. Understand the concept of matrices and their types, including row, column, square, diagonal, and identity matrices.
2. Perform basic operations on matrices such as addition, subtraction, and multiplication.
3. Compute the transpose, determinant, and inverse of a matrix.

4. Solve systems of linear equations using matrix methods, including Cramer's rule and matrix inversion.

10.3 Introduction to Matrices

Typically, a m × n matrix is expressed as follows: [a_11 a_12 ⋯ a_1n a_21 a_22 ⋯ a_2n ⋮ ⋮ ⋮ ⋮ a_31 a_32 ⋯ a_3n]
In a nutshell, A = [aij] mxn represents the matrix above. The elements are the numbers a11, a12, ….. etc., while the (i, j)th element of matrix A = [aij] is the number that appears in the ith row and jth column.

Important Formulas for Matrices

17 If A and B are square matrices of order n, and I_n is a corresponding unit matrix, then

(a) A(adj.A) = | A | In = (adj A) A
(b) | adj A | = | A |n-1 A (adj A) is therefore always a scalar matrix.
(c) adj (adj.A) = | A |n-2 A

(f) adj (AB) = (adj B) (adj A)
(g) adj (Am) = (adj A)m,

(h) adj(kA)=k^{n-1} (adj.A), k ∈R

(i) adj(Am)= (adjA)Adj 0 = 0 (k) m (j) Since A is symmetric, adj A is symmetric as well.

(l) A is diagonal ⇒adj A is also diagonal
(m) A is triangular ⇒adj A is also triangular
(n) A is singular ⇒| adj A | = 0

Types of Matrices

(i) Symmetric matrix: If aij = aji for all i, j, then a square matrix A = [aij] is a symmetric matrix. When aij = -aji, the matrix is skew-symmetric.
(iii) Hermitian matrix with skew-Hermitian:

The Hermitian matrix is A = A^θ, where Aθ stands for conjugate transpose.

-A (skew-Hermitian matrix) = A^θ (iv) matrix orthogonal: if AAT = In = ATA (v) If A2 = A (vi), then the matrix is idempotent. matrix that is involuntary: if A2 = I or A-1 = A (vii) Nilpotent matrix: If p is an integer and Ap = 0, then the square matrix A is nilpotent.

Trace of Matrix

The trace of a square matrix is the sum of the elements on the main diagonal.

(i) tr(λA_ = λ tr(A)

(ii) tr(A + B) = tr(A) + tr(B)

(iii) tr(AB) = tr(BA)

Transpose of Matrix

A matrix that is produced by rearranging its rows into columns or columns into rows from a given matrix A is called a transpose. AT or A' stand for A and. The definition makes it clear that if the order of A is m x n, the order of AT is n x m. This is evident in the transpose of a matrix, for instance.

$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$ is $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$

Properties of Transpose of Matrix

(i) $(A^T)^{T=A}$

(ii) $(A+B)^T = A^T + B^T$

(iii) $(AB)^T = B^T A^T$

(IV) $(kA)^T = k(A)^T$

Problems on Matrices

Illustration 3: If

A = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$

then prove that $(AB)^T = B^TA^T$.

Solution:

By obtaining the transpose of AB, i.e., $(AB)^T$ and multiplying B^T and A^T, we can easily get the result.

Here, AB = $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1(-1) & -2(-1)+ & 3(2) & 1(3)+ & 2(0)+ & 3(4) \\ -4(1) & +2(-1)+ & 5(2) & -4(3)+ & 2(0)+ & 5(4) \end{bmatrix}$$

$$\begin{vmatrix} 9 & 15 \\ 4 & 8 \end{vmatrix}$$

Therefore $(AB)^T$ = $\begin{vmatrix} 9 & 15 \\ 4 & 8 \end{vmatrix}$

$B^T A^T$ = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ -2 & 2 \\ 3 & 5 \end{bmatrix}$

$$\begin{matrix} 1(1) & -1(-2)+ & 2(3) & 1(-4)- & 1(2)+ & 2(5) \\ 3(1) & +0(-2)+ & 4(3) & 3(-4)+ & 0(2)+ & 4(5) \end{matrix}$$

$$\begin{vmatrix} 9 & 4 \\ 15 & 8 \end{vmatrix}$$

=(AB^T)

10.2 Basic operations on Matrices: Addition, Subtraction, multiplication

Addition of matrices

Matrix Operations are the operations that are operated on the matrix. Matrix Operation includes operations such as Addition of Matrix, Subtraction of Matrix, Multiplication of Matrix, etc, and others. These operations are very useful for solving various problems of matrices and help us to find the transpose, inverse, rank, and others of the matrix. These operations help us to combine two or matrices.

What are Matrix Operations?

Matrix operations are techniques used to combine different matrices into a single matrix. Operations like addition, subtraction, and multiplication are straightforward to perform on matrices. These operations are essential for solving matrix problems and for finding the transpose and inverse of a matrix. Various matrix operations that are used to solve matrix problems are,

- Addition of Matrix
- Subtraction of Matrix
- Scalar Multiplication of Matrix
- Multiplication of Matrix

Addition of Matrices

As we add two numbers we can easily add two matrices. The only thing we have to note is that the order of both the matrices that are to be added must be the same. That is to add two matrices we have to make sure that they are of the same order and then each element of the first matrix adds with each element of the second matrix to get a single matrix and thus the addition operation gets completed.

For any matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and matrix B = $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ the sum of the matrix A and matrix B is C matrix then,

C = A+B = $\begin{bmatrix} a+p & b+q \\ c+r & d+s \end{bmatrix}$

Example: Take three matrices A, B, and C of order 2×2, 2×2, and 3×3 respectively.

A = $\begin{bmatrix} \mathbf{2} & \mathbf{9} \\ \mathbf{5} & \mathbf{6} \end{bmatrix}$ **and B =** $\begin{bmatrix} \mathbf{1} & \mathbf{7} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$ **and C =** $\begin{bmatrix} \mathbf{5} & \mathbf{4} & \mathbf{8} \\ \mathbf{2} & \mathbf{3} & \mathbf{7} \end{bmatrix}$

Find the sum of (A+B) and (A+C)

Solution: Matrix A and Matrix B can be easily added as their order is the same. The addition of matrix A and matrix B is found as,

A+B =P = $\begin{bmatrix} 2+1 & 9+7 \\ 5+2 & 6+3 \end{bmatrix}$

P = = $\begin{bmatrix} \mathbf{3} & \mathbf{16} \\ \mathbf{7} & \mathbf{9} \end{bmatrix}$

Matrix A and matrix C can not be added as their order is not the same

Matrix Addition properties

For matrices A, B, and C of the same order, among other properties related to matrix addition, then

- **Commutative Law:** A + B = B + A
- **Associative Law:** (A + B) + C = A + (B + C)
- **Identity of Matrix:** A + O = O + A = A, where O is a zero matrix which is the Additive Identity of Matrix
- **Additive Inverse:** A + (-A) = O = (-A) + A, where (-A) is obtained by changing the sign of every element of A, which is the additive inverse of the matrix.

Subtraction of Matrices

As we add two matrices e can also easily subtract two matrices. The only thing we have to note is that the order of both the matrices that are to be subtracted must be the same. That is to subtract two matrices we have to make sure that they are of the same order and then each element of the first matrix is subtracted with each element of the second matrix to get a single matrix and thus the subtraction operation gets completed.

For any matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and matrix B = $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$the difference of the matrix A and matrix B is C matrix then,

C = A – B = $\begin{bmatrix} a-p & b-q \\ c-r & d-s \end{bmatrix}$

Example: For matrices A and B subtract the matrix B from matrix A

A and B = $\begin{bmatrix} 1 & 7 \\ 2 & 3 \end{bmatrix}$

Solution:

Matrix A and Matrix B can be easily subtracted as their order is the same. The subtraction of matrix A and matrix B is found as,

A – B = P = $\begin{bmatrix} 2-1 & 9-7 \\ 5-2 & 6-3 \end{bmatrix}$

P = = $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$

Scalar Multiplication of Matrices

For any matrix A = $[a_{ij}]_{m\times n}$ if we multiply the matrix A with any scaler (say k) then the scaler is multiplied by each element of the matrix and this is called the scalar multiplication of matrices.

For any matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if it is multiplied by any scaler k then,

kA = $\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

Properties of Scalar Multiplication

For any matrices, A and B of the same order and λ and µ are any two scalars, then,

- $\lambda(A + B) = \lambda A + \lambda B$
- $(\lambda + \mu)A = \lambda A + \mu A$

- λ(μA) = (λμA) = μ(λA)
- (-λA) = -(λA) = λ(-A)

Multiplication of Matrix

The process that enables us to multiply two matrices is called matrix multiplication. Not all matrices can be multiplied, and this is not the same as algebraic multiplication. Only those matrices can be multiplied where the number of columns in the first is equal to the number of rows in the second, i.e for matrix Am×n and matrix Bn×p the multiplication is possible for any other matrices where the column of the first matrix is not equal to the row in the second matrix the multiplication is not possible.

Transpose Operation of a Matrix

The transpose operation of a matrix is used to obtain the transpose of a given matrix. The transpose of a matrix is formed by converting its rows into columns and its columns into rows.. Suppose we have a matrix A of order m×n such that A =[ij]m×n then the transpose of matrix A is represented as (A)T and its value is,

$(A)^T$ = [ji]n×m

Inverse Operation of a Matrix

For any matrix A its inverse is found only when A is a square matrix and its determinant is equal to 1, i.e.

A = [ij]n×n and |A| = 1

Now the inverse of a matrix A is a matrix that on multiplying with the matrix A results in the identity matrix. It is represented as (A)-1, and the inverse operation of the matrix is an operation that helps us to find the inverse of the matrix. For any square matrix A we know that,

A × $(A)^{-1}$ = I

Where ''I'' is the identity matrix

Solved Examples on Matrix Operations

Example 1: Find the sum of matrix A and B when,

A = $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$

B = $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$

Solution:

Matrix A and Matrix B can be easily added as their order is the same. The addition of matrix A and matrix B is found as,

A + B = P= $\begin{bmatrix} 1+2 & 3+4 \\ 5+6 & 7+8 \end{bmatrix}$

P = $\begin{bmatrix} 3 & 7 \\ 11 & 15 \end{bmatrix}$

Example 2: Find (A – B) when,

A = $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$

Solution: Matrix A and Matrix B can be easily subtracted as their order is the same. The value of (A-B) is found as,

(A -B)+ P = $\begin{bmatrix} 2-1 & 4-3 \\ 6-5 & 8-7 \end{bmatrix}$

P = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

10.3 Determinants and inverse of a matrix

In linear algebra, determinants and matrices are used to solve linear equations by applying Cramer's rule to a set of non-homogeneous linear equations. Determinants are calculated only for square matrices. A determinant is referred to as singular if its value is zero, and as unimodular if its value is one.

Properties of Determinant

- If I_n is the identity matrix of the order nxn, then det(I) = 1
- If the matrix M^T is the transpose of matrix M, then det (M^T) = det (M)
- If matrix M^{-1} is the inverse of matrix M, then det (M^{-1}) = 1/det (M) = det $(M)^{-1}$
- If two square matrices M and N have the same size, then det (MN) = det (M) det (N)

- If matrix M has a size axa and C is a constant, then det (CM) = C^a det (M)
- If X, Y, and Z are three positive semidefinite matrices of equal size, then the following holds true along with the corollary det (X+Y) ≥ det(X) + det (Y) for X,Y, Z ≥ 0 det (X+Y+Z) + det C ≥ det (X+Y) + det (Y+Z)
- In a triangular matrix, the determinant is equal to the product of the diagonal elements.
- The determinant of a matrix is zero if all the elements of the matrix are zero

Inverse of a matrix

A matrix is said to be its inverse if it produces an identity matrix when multiplied by the original matrix. The inverse of any square matrix A is represented by the symbol A-1. After dividing the adjugate of a matrix by its determinant, the inverse of the matrix is obtained.

Where I is the identity matrix,

$A^{-1} = \frac{1}{|A|}$ Adj A

What is an Inverse Matrix?

The inverse of a matrix is another matrix that, when multiplied by the given matrix, yields the **multiplicative identity**.

For matrix A and its inverse of A-1, the identity property holds.

A.A-1 = A-1A = I

Terms Related to Inverse of a Matrix

The terminology listed below can help you grasp the inverse of a matrix more clearly and easily.

Terms	Definition	Formula/Process	Example with Matrix A

Terms	Definition	Formula/Process	Example with Matrix A
Minor	The minor of an element in a matrix is the determinant of the matrix formed by removing the row and column of that element.	For element aij , remove the ith row and jth column to form a new matrix and find its determinant.	Minor of *a11* is the determinant of A=[5667]*A*=[5667]
Cofactor	The cofactor of an element is the minor of that element multiplied by **(-1)i+j** , where i and j are the row and column indices of the element.	Cofactor of aij = (-1)i+j Minor of aij	Cofactor of *a11* = **(-1)1+1** × Minor of *a11* = Minor of *a11*
Determinant	The determinant of a matrix is calculated as the sum of the products of the elements of any	For a row (or column), sum up the product of each element and its cofactor.	Determinant of A = *a11* × Cofactor of *a11* +*a12* × Cofactor of *a12* +*a13* × Cofactor

Terms	Definition	Formula/Process	Example with Matrix A
	row or column and their respective cofactors.		of *a13*.
Adjoint	The adjoint of a matrix is the transpose of its cofactor matrix.	Create a matrix of cofactors for each element of the original matrix and then transpose it.	Adjoint of A is the transpose of the matrix formed by the cofactors of all elements in A.

Singular Matrix

A singular matrix is one whose determinant value is zero; that is, any matrix A is a singular matrix if $|A| = 0$. There is no inverse of a singular matrix.

Non-Singular Matrix

A non-singular matrix is one whose determinant value is non-zero; that is, any matrix A is a non-singular matrix if $|A| \neq 0$. There is an inverse of a non-singular matrix.

Identity Matrix

A square matrix in which all the elements are zero except for the principal diagonal elements is called the identity matrix. It is represented using I. It is the identity element of the matrix as for any matrix A,

$A \times I = A$

An example of identity matrix is,

$I_{3\times3} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

This is an identity matrix of order 3×3

How to Find Inverse of a Matrix?

There are Two-ways to find the Inverse of a matrix in mathematics:

- **Using Matrix Formula**
- **Using Inverse Matrix Methods**

Inverse of a Matrix Formula

The inverse of matrix A, that is A-1 is calculated using the inverse of matrix formula, which involves dividing the adjoint of a matrix by its determinant.

$A^{-1} = \frac{AdjA}{|A|}$

Where ,

Adj A = adjoint of the matrix A, and

$|A| =$ determinant of the matrix A.

To find inverse of matrix using inverse of a matrix formula, follow these steps.

Step 1: Determine the minors of all A elements.

Step 2: Next, compute the cofactors of all elements and build the cofactor matrix by substituting the elements of A with their respective cofactors.

Step 3: Take the transpose of A's cofactor matrix to find its adjoint (written as adj A).

Step 4: Multiply adj A by the reciprocal of the determinant of A.

Now, for any non-singular square matrix A,

$\boldsymbol{A^{-1}}$**= 1 / |A| × Adj (A)**

Matrix inverse solved examples

Example 1: Find the inverse of the matrix A=$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$ using the formula.

Solution:

We have,

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$$

Find the adjoint of matrix A by computing the cofactors of each element and then getting the cofactor matrix's transpose.

Adj A = $\begin{bmatrix} -2 & -9 & 5 \\ 0 & 6 & -3 \\ 1 & 0 & -1 \end{bmatrix}$

Find the value of the determinant of the matrix.

|A|= 2(4–6) – 3(4–4) + 1(3–2) = -3

So, the inverse of the matrix is,A^{-1} = $\frac{1}{-3}\begin{bmatrix} -2 & -9 & 5 \\ 0 & 6 & -3 \\ 1 & 0 & -1 \end{bmatrix}$

= $\begin{bmatrix} 2/3 & 3 & -5/3 \\ 0 & -2 & 1 \\ -1/3 & 0 & 1/3 \end{bmatrix}$

Example 2: Find the inverse of the matrix A=\bold{ using the formula.}

$$\begin{bmatrix} 6 & 2 & 3 \\ 0 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix}$$

Solution: we have,

A = $\begin{bmatrix} 6 & 2 & 3 \\ 0 & 0 & 4 \\ 2 & 0 & 0 \end{bmatrix}$

Find the adjoint of matrix A by computing the cofactors of each element and then getting the cofactor matrix's transpose.

Adj A = $\begin{bmatrix} 0 & 0 & 8 \\ 8 & -6 & -24 \\ 0 & 4 & 0 \end{bmatrix}$

Find the value of determinant of the matrix.

|A| = 6(0–4) – 2(0–8) + 3(0–0)

= 16

So, the inverse of the matrix is,

$$A^{-1} = \frac{1}{16}\begin{bmatrix} 0 & 0 & 8 \\ 8 & -6 & -24 \\ 0 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1/2 \\ 1/2 & -3/8 & -3/2 \\ 0 & 1/4 & 0 \end{bmatrix}$$

10.4 applications of matrices in solving linear equations

Applications of Matrices and Determinants:

One of the key applications of matrices and determinants is solving linear equations involving two or three variables. They are also utilized to determine the consistency of a system, identifying whether it is consistent or inconsistent. This is considered one of the most practical uses of matrices and determinants. In physics, matrices and determinants are utilized to study electrical circuits, convert electrical energy through resistors, and analyze optics. Additionally, they assist in calculating battery power outputs, understanding quantum mechanics, and other related applications.

Applications of Matrices and Determinants

- Systems of Linear Equations
- Computer Graphics
- Physics and Engineering
- Cryptography
- Economics and Statistics

Using a matrix's inverse to solve a system of linear equations

The inverse of a matrix can be used to determine the solution of a system of linear equations. Let's look at the equations:

$a_1x + b_1y + c_1z = d_1$

a_2x r+ $b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

these equations can be represented using a matrix as follows.

$a_1x + b_1y + c_1z = d_1$

a_2x r+ $b_2y + c_2z = d_2 \quad = \begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix}$

$a_3x + b_3y + c_3z = d_3$

Further, this can be written as

$$\begin{bmatrix} a_1x + & b_1y + & c_1\,z \\ a_2x + & b_2y + & c_2z \\ a_3x + & b_3y + & c_3z \end{bmatrix} = \begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix}$$

Further, this can be written as

$$\begin{bmatrix} a_1 + & b_1 + & c_1 \\ a_2 + & b_2 + & c_2 \\ a_3 + & b_3 + & c_3 \end{bmatrix} = \begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix}$$

Further, this system can be written as

$AX = B$

where matrix A contains coefficients of unknown variables.

$$A = \begin{bmatrix} a_1 + & b_1 + & c_1 \\ a_2 + & b_2 + & c_2 \\ a_3 + & b_3 + & c_3 \end{bmatrix}$$

Matrix X is a column matrix that contains the unknown variables

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Matrix B is also a column matrix, and it contains the constants.

B = $\begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix}$

a system of linear equations can be converted into the form of a matrix which can be written as: AX = B

If A is a non-singular matrix then A^{-1} exists.

Multiplying by A-1 on both sides

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

This yields a unique solution for the unknown variables, as a non-singular matrix always has a unique inverse.

If A is a singular matrix then A^{-1} doesn't exist. In this case |A| = 0, so you will have to calculate (adj A)B.

1. If (adj A)B ≠ O, In that case, no solution exists for the system of linear equations, making the system inconsistent.

2. If (adj A)B = O, In this scenario, there will either be no solution or an infinite number of solutions to the system of linear equations. Therefore, the system may be inconsistent if no solution exists or consistent if it has infinitely many solutions.

Sample Problems – Applications of Matrices and Determinants

Question 1. Solve the following linear equations using matrix

2x + y = 3

2x + 3y = 6

The above system of linear equations can be written in the form of AX = B where A is the matrix of coefficients, X is the matrix of unknown variables and B is the matrix of constants**.**

A = $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

X = $\begin{bmatrix} x \\ y \end{bmatrix}$

B = $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$

First find out the |A|

as you can see |A| = 4 ≠ 0. Hence, the system of equations is consistent and will possess an unique solution and the solution can be found out using X = $A^{-1}B$

$A^{-1} = \frac{adjA}{|A|}$

A^{-1} = ¼ $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

X = $A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix}$ =¼ $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix}$= ¼ $\begin{bmatrix} 9 - & 6 \\ -6 + & 12 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = 1/4\begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

From here, you can conclude that

x = 3/4

and y = 6/4

Question 3. Solve the system of linear equations, using matrix method

x – y + 2z = 7

3x + 4y – 5z = -5

2x – y + 3z = 12

AX = B is one way to express the aforementioned system of equations, where

A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$

X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

B = $\begin{bmatrix} 7 \\ -5 \\ -12 \end{bmatrix}$

Now check determinant of A

|A| = 1 * (12 – 5) + 1 * (9 + 10) + 2 * (-3 – 8)

|A| = 7 + 19 – 22 = 4

|A| ≠ 0

Hence, its inverse exists and hence there exists a unique solution that can be found out by X = $A^{-1}B$

$$A^{-1} = \frac{adjA}{|A|}$$

$$A^{-1} = 1/4\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

X = A^{-1}B

= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = $1/4\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ -12 \end{bmatrix}$

= $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1/4\begin{bmatrix} 49 - & 5 & -36 \\ -133 & +5 & 132 \\ -77 & +5 & +84 \end{bmatrix}$

= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$1/4$\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$

= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

From here, you can see that x =2, y = 1, and z = 3.

Hence, x = 2, y = 1, and z = 3.

10.5 check your progress

Classify the following matrices:

(i)$\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

(iv) $\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$ (v)$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Q. find the values of x and y if ;

(i)$\begin{bmatrix} 2x + & y \\ 3x - & 2y \end{bmatrix}$=$\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

(ii) $\begin{bmatrix} 3x + y - & y \\ 2y - x & 3 \end{bmatrix}$= $\begin{bmatrix} 1 & -2 \\ -5 & 3 \end{bmatrix}$

(iii) find the values of x,y,a and b if

$\begin{bmatrix} x - 2 & y \\ a = 2b & 3a - b \end{bmatrix}$= $\begin{bmatrix} 3 & -1 \\ 5 & 1 \end{bmatrix}$

Q. let A = $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 4 \\ 3 & -1 \end{bmatrix}$, calculate A+ B

Q. Find the determinant of the matrix A = $\begin{bmatrix} 4 & 5 \\ 3 & 7 \end{bmatrix}$

Q. find the determinant of the following linear equation in three variables

(i). 2x + y + z =1

X – 2y – z = 3/2

3y -5z = 9

(ii) x + y+ z =4

2x+ y-3z = 0

X + y + z =2

10.8:Unit Summary:

This unit covers the fundamental operations on matrices, including addition, subtraction, scalar multiplication, and matrix multiplication. It also explores advanced topics such as the calculation of the **determinant**, **transpose**, and **inverse** of a matrix.

Matrix algebra is widely used to solve systems of linear equations through methods like **Gaussian elimination**, **Cramer's Rule**, and **matrix inversion**. It also introduces concepts like the rank of a matrix and eigenvalues, which are essential in higher mathematics. The unit highlights practical applications of matrices in various fields, such as computer graphics, engineering, economics, and data analysis. By mastering matrix algebra, students gain the ability to model and solve complex problems involving linear systems and transformations.

10.9 check your progress

Q. find the values of x and y if ;

(i)$[2x+\ y\ 3x-\ 2y\ ]$=$[5\ 4\ ]$

(ii) $[3x+y-\ y\ 2y-x\ 3\ ]$= $[1\ -2\ -5\ 3\ ]$

(iii) find the values of x,y,a and b if

$[x-2\ y\ a=2b\ 3a-b\ ]$= $[3\ -1\ 5\ 1\ ]$

Q. let A = $[1\ 2\ 3\ 1\ ]$, B = $[1\ 4\ 3\ -1\ ]$, calculate A+ B

Q. Find the determinant of the matrix A = $[4\ 5\ 3\ 7\ ]$

Q. find the determinant of the following linear equation in three variables

(i). 2x + y + z =1

X – 2y – z = 3/2

3y -5z = 9

(ii) x + y+ z =4

2x+ y-3z = 0

Unit 11: Introduction to Data Analysis

11.1 Introduction

Data analysis is the process of systematically applying statistical and logical techniques to describe, summarize, and evaluate data. In today's world, data is generated in massive volumes, and understanding how to manage, interpret, and analyze this data is essential in various domains, including business, healthcare, social sciences, and engineering.

The objective of this unit is to introduce the concepts and methods used in data analysis, particularly the ways in which data can be organized and classified for easier understanding. By the end of this unit, students should be familiar with the processes involved in grouping data, constructing frequency distributions, tabulating data, and using graphical methods to represent data.

11.2 Unit Objectives

The primary objectives of this unit are as follows:

1. **Understand the importance of data organization**: Learn how organizing data can simplify analysis and decision-making.

2. **Understand the importance of data organization** Identify various classification methods and their relevance.

3. **Learn how to group data and create frequency distributions**: Discover how data can be grouped for easier interpretation.

4. **Master tabulation and graphical representation techniques**: Learn how to present data in tables and graphs for clear, concise communication.

5. **Apply data organization in decision-making**: Understand how well-organized data is crucial in effective decision-making processes.

6. **Familiarize with real-world applications of data analysis**: Understand the impact of data organization in everyday decisions and business operations.

11.3 Organizing and Classifying Data: Methods and Importance

Sorting and categorizing data to facilitate comparison and interpretation is the first step in data analysis. Proper organization of data is key to drawing meaningful conclusions.

Methods of Organizing Data:

Tabular Organization: For convenience, data is organized in rows and columns.

Categorization: Data is grouped based on certain characteristics (e.g., age groups, income levels, etc.).

· **Sorting**: : Data is arranged in ascending or descending order based on certain variables.

- **Importance of Data Organization**:
- **Efficiency**: Organized data can be processed more efficiently.
- **Accuracy**: Classification ensures that data is analyzed in the correct context. Interpretability: Proper organization helps present complex data in a way that is easy to understand.

11.4 Grouping Data and Frequency Distributions

Grouping data into categories is essential for making sense of large datasets. A frequency distribution is a way to show how often each data value occurs, which helps in identifying patterns, trends, and outliers.

- **Grouping Data**:
- **Class Intervals**: Group data into intervals (e.g., 0–10, 11–20) to summarize large datasets.
- **Class Width**: The difference between the upper and lower boundaries of a class interval.
- **Class Boundaries**: The exact range of values that belong in each class.
- **Frequency Distribution**:

· A **frequency distribution** table lists the data values or intervals alongside the number of occurrences (frequency). It is a helpful tool to visualize how data is distributed across different intervals.

Cumulative Frequency: Shows the running total of frequencies.

1. Create a grouped frequency distribution table with equal-width class intervals beginning with 25–30, 30–35, and so forth. See the range of weekly pocket expenses as well.

Solution: The following table represents the given data:

Weekly expenses (in $)	**Number of students**
25-30	2
30-35	4
35-40	10
40-45	4

45-50	5
Total	**25**

In the given data, the smallest value is 26 and the largest value is 49. So, the range of the weekly pocket expenses = 49 - 26 = $23.

Ms. Kate collected data of the weights of the students of her class and prepared the following table:

Weight (in lbs)	64−6964−69	70−7570−75	76−8176−81
Number of students	88	1515	2525

Mean of Grouped Data

Consider the height of 5 students:

144 cm, 158 cm, 153 cm, 156 cm, 162 cm.

Then the arithmetic mean of the height of these five students is given by:

Mean height = 154.6 cm

In general, if

Or = /n

Here, the symbol denotes

Mean of Grouped Data: Direct Method

If are observations having respective frequencies

This means occurs times, occurred times and so on. Then,

Sum of the values of observations = ...

and sum of frequencies =

=

=

1) The following data represents which numbers are rolled with a standard six-sided dice:

1, 3, 5, 3, 2, 1, 2, 5, 6, 4, 5, 2, 6, 1, 4, 3, 6, 1, 2, 4, 6, 1, 3, 1, 3, 5, 6

Number xi	**Frequency** fi	fixi
1	6	6
2	4	8
3	5	15
4	3	12
5	4	20

6	5	30

= 91/27 = 3.37

Sometimes, data is so large that it is very difficult to find its mean. In this situation, we make groups of the data with a suitable class interval. Then, we find the classmark which is given:

In this method, while allotting frequencies to each class interval, it should be noted that frequency equal to upper-class interval should not be put in the same class interval i.e., it should not be put in the next class interval. For example, in the class interval 25-35, 35 should not be put in the next class interval, i.e., 35-45 class interval.

2) In Tarun's school, there are 25 teachers. Each teacher travels to school every morning in his or her own car. The distribution of the driving times (in minutes) from home to school for the teachers is shown in the table below:

Driving time (in minutes)	**Number of Teachers**
0-10	3
10-20	10
20-30	6

30-40	4
40-50	2

To better represent the problem and its solution, a table can be drawn as follows:

Driving time (in minutes)	**Mid-values** xi	**Number of teachers** fi	fixi
0-10	5	3	15
10-20	15	10	150
20-30	25	6	150
30-40	35	4	140
40-50	45	2	90
		=25	

We have,

= 545/25 = 21.8

Each teacher spends a mean time of 21.8 minutes driving from home to school each morning.

11.5 Tabulation of Data

Data tabulation and graphical representation are essential for summarizing and presenting data in a visually appealing and easy-to-understand format.

Tabulation: Data can be organized into frequency tables, which count the number of times each value occurs and classify the data into groups or intervals.

Graphical Representation:

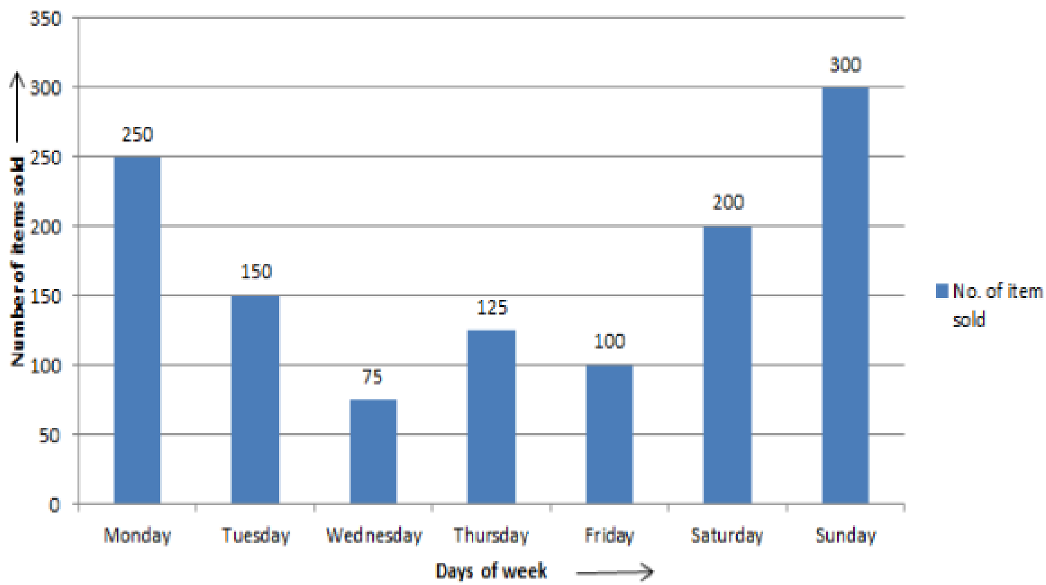
· **Pie Charts**: Used for showing proportions or percentages of a whole, often used in categorical data.

Line Graphs: beneficial for displaying data trends over time.

Importance: Graphical representations make complex data more accessible and allow patterns to be easily spotted.

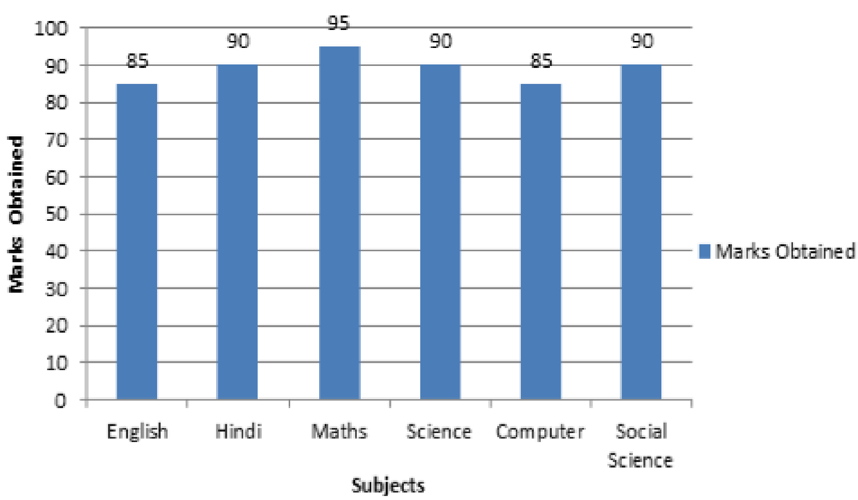
Examples

The bar chart for the example of the number of items sold during the sale is



Question 1: What is the frequency of 20 marks if 15 students obtained 20 marks in a test?
Answer : The frequency of 20 marks is 15.

Problem: From the given bar-graph answer the following questions:



a. In which subject the student scored the highest?

b. What is the difference between the highest and the lowest marks?

c. In how many subjects, the student got less than 90 marks?

d. What is the total mark scored by the students?

Solution: From the bar graph,

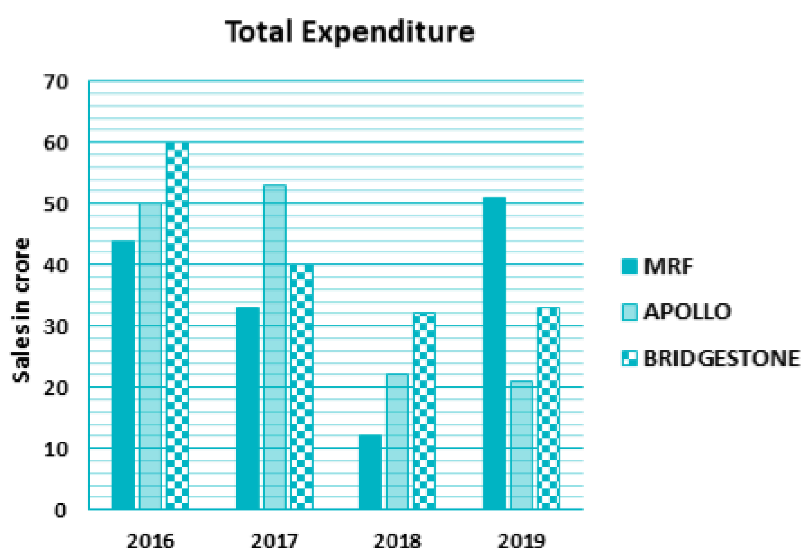
a. The student scored the highest marks in maths.

b. The difference between the highest and the lowest marks = 95 − 90 = 10.

c. Two subjects.

d. The total marks = 85 + 90 + 95 + 90 + 85 + 90 = 535.

The below Bar Graph shows the sales of different brands of TYRE



Q1. Find the ratio of sales of MRF in 2016 and Apollo in 2019.

1. 44:21

2. 204:504

3. 1:2

4. 200:400

5. None of these

Solution: Given:

Sales of MRF tyres in 2016 = 44 crore

Sales of Apollo tyres in 2019 = 21 crore

Calculation:

Sales of MRF tyres in 2016 44 crore =

Sales of Apollo tyres in 2019 = 21 crore

Resultant Ratio = **44: 21**

Q2. Find the average sales of 2018 and 2019 including all the brands

1. 85 Cr

2. 85.5 Cr

3. 80 Cr

4. 45 Cr

5. 90 Cr

Answer (Detailed Solution Below): Option B: **85.5 Cr**

Solution: Given:

Sale in 2018 (12+22+32) Crores

Sale in 2019 = (51+ 21 +33) Crores

Concept Used:

Average = sum / total observation

Calculation:

Sale in 2018 (12+22+32) Crores

Sale in 2019 (51+21+33) Crores

The average is = (66+105)/2

The average of both years is 171/2 = **85.5 crores**

Q3. What are the average sales of tyres in the year 2019

1. 40 Cr
2. 50 Cr.
3. 35 Cr
4. 44 Cr
5. None of these

Solution: Given:

Sale of tyres in 2019 for all the brands

Sales of MRF in 2019 = 51 crore

Sales of APOLLO in 2019 = 21 crore

Sales of Bridgestone in 2019 = 33 crore

Concept Used:

Average = sum / total observation

Calculation:

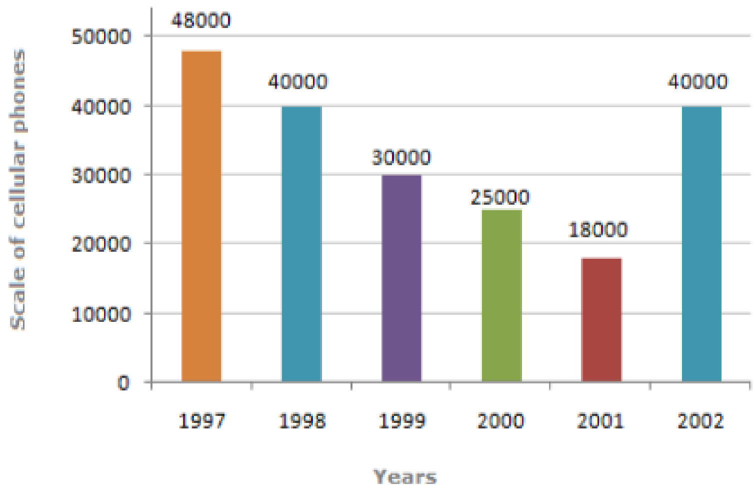
Sales of BRIDGESTONE in 2019 = 33 crores

Average = (51 + 21 + 33)/3

Average = (105)/3 crore

The average is 35 crores

Study the bar chart and answer the questions.



1. The difference in the sales of cellular phones for the years 1997 and 1999 is?

1. 500 units
2. 1,000 units
3. 5,000 units
4. 18,000 units

Solution: The required answer is got by 48,000 – 30,000 = 18,000.

2. The two years between which the rate of change of cellular phones is minimum are?

1. 1997 and 1998
2. 1999 and 2000
3. Both options (A) and (B)
4. 2001 and 2002

Solution: The lowest rate of change for

For years 1997 and 1998 = ((48000 – 40000) / 40000) x 100 = 20%

For years 1999 and 2000 = ((30000 – 25000) / 25000) x 100 = 20%

is exhibited by both options (A) and (B).

3. The sum of sales of cellular phones in the years 1999 and 2001 is equal to that in?

1. 1997
2. 1998
3. 2000
4. 2002

Solution: The sum of sales in the two years is 30,000 + 18,000 = 48,000, which is the sales value for 1997.

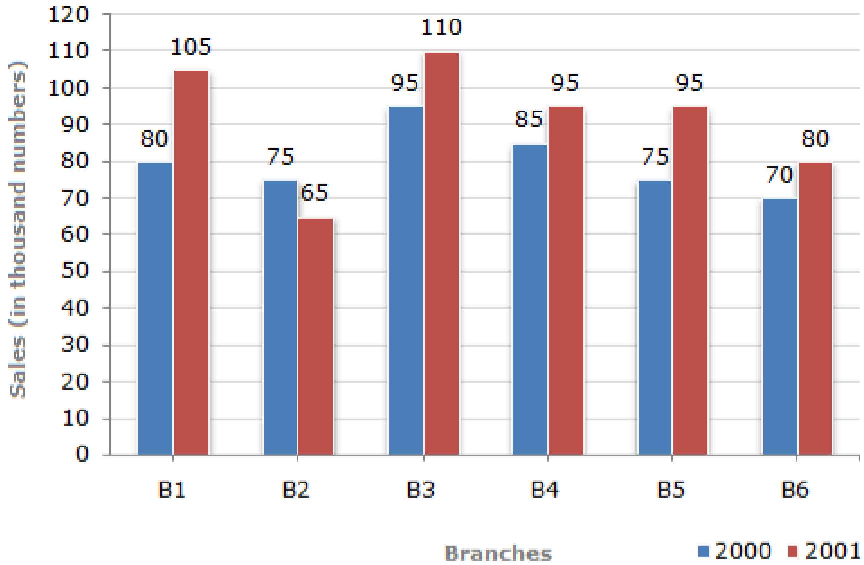
4. The percentage increases in sales from 2001 to 2002 was?

1. 115 %
2. 128 %
3. 122 %
4. 118 %

Solution: The percentage increase exhibited is

((40 – 18)/18) x 100 = 122 % approximately.

Q.The bar graph below illustrates the book sales (in thousands) from six branches (B1, B2, B3, B4, B5, and B6) of a publishing company for the years 2000 and 2001.



1. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?

1. 2:3
2. 3:5
3. 4:5
4. 7:9

Solution: Required ratio = (75+65)/(85+95) = 140/180 = 7/9

2. Total sales of branch B6 for both years is what percent of the total sales of branch B3 for both **years?**

1. 68.54%
2. 71.11%
3. 73.17%
4. 75.55%

Solution: Required percentage = (70+80)/(95+110) x 100 = 150/205 x 100 = 73.171%

3. What percent of the average sales of branches B1, B2, and B3 in 2001 is the average sales of branches B1, B3, and B6 in 2000?

1. 75%
2. 77.5%
3. 82.5%
4. 87.5%

Solution: Average sales (in thousand numbers) of branches B1, B3, and B6 in 2000

=1/3 x (80 + 95 + 70) = (245).

Average sales (in thousand number) of branches B1, B2 and B3 in 2001

=1/3 x (105 + 65 + 110)=(280).

Therefore Required percentage = [245/3 / 280/3 x 100]% = (245/280 x 100)% = 87.5%.

4. What is the average sales of all the branches (in thousand numbers) for the year 2000?

1. 73
2. 80
3. 83
4. 88

Solution: Average sales of all the six branches (in thousand numbers) for the year 2000

=1/6 x[80 + 75 + 95 + 85 + 75 + 70]= 80.

5. Total sales of branches B1, B3, and B5 together for both years (in thousand numbers) is?

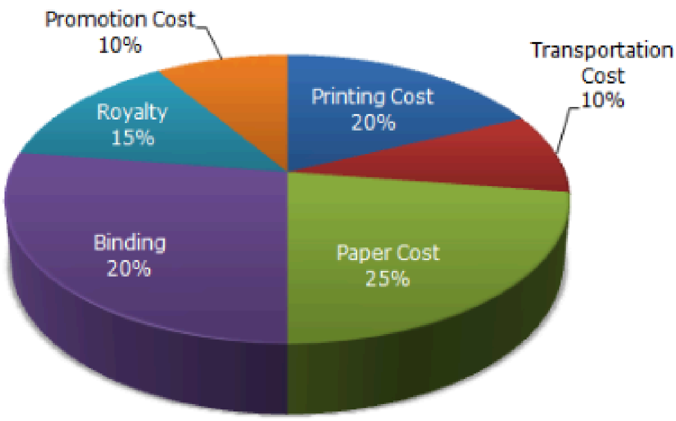
1. 250
2. 310
3. 435
4. 560

Solution: Total sales of branches B1, B3, and B5 for both the years (in thousand numbers)

= (80 + 105) + (95 + 110) + (75 + 95) = 560.

Pie charts

The pie-chart below illustrates the percentage breakdown of the expenses involved in publishing a book. Study the chart and answer the related questions.



1.What amount of royalty will be due for these books if the publisher must pay Rs. 30,600 for printing a specific number of them?

Rs. 19,450

a) Rs. 21,200

b) Rs. 22,950

c) Rs. 26,150

Solution

Let the amount of royalty to be paid for these books be Rs.r

Then, 20:15 = 30600:r => r = $\left(\frac{30600 \times 15}{20}\right)$

= Rs. 22,950

2. What is the sector's central angle that corresponds to the royalty expenditure?

a) 15°

b) 24°

c) 54°

d) 48°

Solution:

Central angle corresponding to Royalty is = ($15\%\ of\ 360^{\circ}$)

$$= \frac{15}{100} \times 360^{\circ}$$

54°

3. The price of the book is marked 20% above the C.P. If the marked price of the book is Rs. 180, then what is the cost of the paper used in a single copy of the book?

a) Rs. 36

b) Rs. 37.50

c) Rs. 42

d) Rs. 44.25

Clearly, marked price of the book = 120% of C.P.

Also, cost of paper = 25% of C.P

Let the cost of paper for a single book be Rs. *n*.

Then, 120:25 = 180 : n => n = Rs.$\frac{25 \times 180}{120}$

= Rs. 37.50

4. If 5500 copies are published and the transportation cost on them amounts to Rs. 82500, then what should be the selling price of the book so that the publisher can earn a profit of 25%?

a) Rs. 187.50

b) Rs. 191.50

c) Rs. 175

d) Rs. 180

For the publisher to earn a profit of 25%, S.P. = 125% of C.P.

Also Transportation Cost = 10% of C.P.

Let the S.P. of 5500 books be Rs. *x*.

Then, 10:125 = 82500: x

=> x = Rs.$\left(\frac{125\times82500}{10}\right)$

= Rs. 1031250.

Therefore S.P. of one book = Rs.$\frac{1031250}{5500}$ = Rs. 187.50

5. Royalty on the book is less than the printing cost by:

a) 5%

b) 20%

c) 25%

d) $33\frac{1}{5}$%

Solution: Printing Cost of book = 20% of C.P.

Royalty on book = 15% of C.P.

Difference = (20% of C.P.) - (15% of C.P) = 5% of C.P.

Therefore percentage difference = $\frac{Difference}{printing\ cost}\times 100\%$

$$= \frac{5\%\ of\ C.P.}{printing\ cost}\times 100\% = 25\%$$

Line graphs

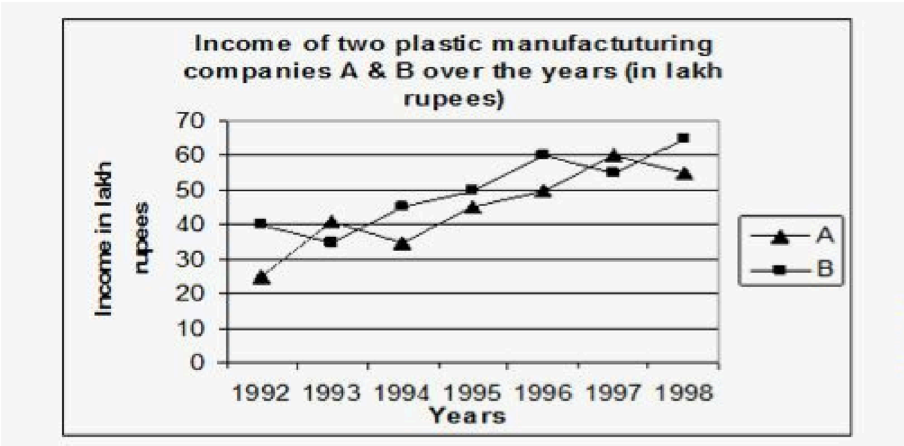
What is a Line Graph?

Line graphs are a type of graph that shows the relationship between two quantitative variables by connecting them with a straight line. The most common use for line graphs is to show the change in one variable over time.

Line graphs are used in a variety of fields such as science, engineering, business and finance. Line graphs can be either horizontal or vertical depending on the variables being plotted. For example, a positive sloping line would indicate an increasing trend and would be helpful for predicting the future value of the data point.

The following graph represents the profit percentage earned by 2 companies A and B on their investment. [Hint: Revenue = Investment + Profit]

DIRECTIONS for questions 1-5: Study the following graph carefully and answer the questions given below (Profit is taken as the % of expenditure.)



Example 1. What would be the ratio of income of company B in 1996 to the income of company A in 1993?

A. 9 : 10

B. 10 : 9

C. 3 : 2

D. 15 : 13

Solution:

Income of Company B in 1996 = 60 lakh. Income of company A in 1993 = 40 lakh. Required ratio = 60 / 40 = 3:2

Example 2. If the expenditure of company B in 1997 is Rs. 50 lakhs, the percent profit earned by both the companies A and B in 1997 is equal, then what is the amount of profit earned by company A in 1997 (approximately)?

A. Rs. 5 lakhs

B. Rs. 4.5 lakhs

C. Rs. 5.5 lakhs

D. Rs. 6.2 lakhs

Income of company B in 1997 = 55 lakh. Expenditure of company B in 1997 = 50 lakh. % profit = 10%
Expenditure = 60 / 1.1 = 54.5 ⇒ amount of profit = 60 – 54.5 = 5.5 lakh

Example 3. If in the year 1995, company 'B' had a profit of 25%, what approximately was its expenditure in the year 1995 ?

A. Rs. 22 lakhs

B. Rs. 29 lakhs

C. Rs. 40 lakhs

D. Rs. 27 lakhs

Solution:

For company 'B' profit = 25%. Income of company B = 50 lakh in 1995
Required expenditure = 50 / 1.25 = 40 lakh

Example 4. The average income of company 'B' per year is what percentage of the average income of company 'A' per year?(approximately)

A. 70%

B. 110%

C. 113%

D. 90%

Solution:

Average income of company A = (25 + 40 + 35 + 45 + 50 + 60 + 55) / 7 = 44.285
Average income of company B = (40 + 35 + 45 + 50 + 60 + 55 + 65) / 7 = 50
Required %age = 50 / 44.285 x 100 = 112.9% ≅ 113%

Example 5. Income of company 'A' in 1996 is what percent of income of company 'B' in 1992?

A. 75%

B. 63.64%

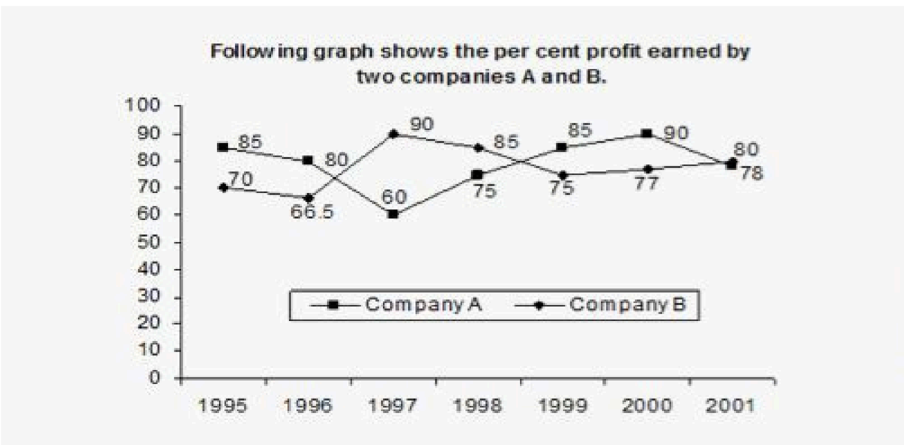
C. 133.33%

D. 125%

Solution:

Income of company B in 1992 = 40 lakh. Income of company A in 1996 = 50 lakh
And 50 lakh is 125% of 40 lakh.

DIRECTIONS *for question 6-10:* Study the following graph carefully and answer accordingly.Following graph shows the percent profit earned by two companies A and B on their investments.(Revenue = Investment + Profit)



Example 6.Revenue of company B in 2000 was Rs.1239 lakhs. What was the investment in that year (in Rs lakhs) of company B?

A. 700

B. 800

C. 650

D. 193.03

Investment of company A in 2000 = 1239 x 100 / 177 = Rs.700 lakh

Example 7. Investment of company B in 1998 was 20% more than that in the previous year. Profit in 1998 of company B was what per cent of its profit in 1997?

A. 10%

B. 102(2/3)%

C. 106(2/3)%

D. None of these

Let us assume the investment of Company B in 1997 is Rs. 100. Investment of company B in 1998 was 20% more than that in the previous year.
So the investment of Company B in 1998 is Rs. 120.
Profit in 1997 of company B= 90 % of 100 = Rs. 90
Profit in 1998 of company B= 85% of 120 = Rs. 102.
Required % = 102 / 90 x 100 = 113 (1/3)%

Example 8. In which of the following years is the ratio of investment and profit maximum for company A?

A. 2001

B. 1995

C. 1998

D. 2000

Quicker Method: The ratio of investment and profit will be maximum when percentage profit is minimum. Therefore, the ratio of investment and profit will be maximum in 2001.

Example 9. If the revenue of company B in 1996 was same as the revenue of company B in 1999, what would be the ratio of investment of company B in 1999 to the investment of company B in 1996?

A. 9 : 10

B. 10 : 9

C. 13 : 15

D. 15 : 13

Solution:

Required ratio = (100 / 185) x (166.5 / 100) = (333 / 370) = (37*9 / 37*10) = 9:10

Example 10. In which of the following years was the investment minimum for company B?

A. 1995

B. 1999

C. 2000

D. Can't be determined

Solution: cannot be determined.

11.6 Practical Applications of Data Organization in Decision Making

Effective data organization is essential for decision-making, particularly in the fields of business and policymaking. The following are some ways that well-organized data facilitates decision-making:

1. **Business Analysis**: Companies use sales data, customer feedback, and market trends to make strategic decisions. Data tables, frequency distributions, and graphs help visualize trends and forecast future developments.

2. **Healthcare**: Organizing patient data helps in tracking symptoms, treatments, and outcomes, leading to more accurate diagnoses and treatment plans.

3. **Government**: Governments use statistical data to make decisions about resource allocation, urban planning, and policy implementation. The classification of census data is an example of how data organization guides decisions on infrastructure and public services.

4. **Education**: Schools and universities use student performance data to evaluate teaching strategies and improve educational outcomes.

5. **Market Research**: Companies rely on organized data to understand consumer behavior, identify market needs, and develop targeted advertising strategies.

11.7 Unit Summary

This unit introduces the fundamentals of data analysis, focusing on the essential steps of organizing, classifying, grouping, and presenting data. We explored different techniques, including tabulation, frequency distributions, and graphical representations. The unit also emphasized the importance of data organization in decision-making across various industries, demonstrating its practical relevance in real-world applications.

11.8 check your progress

Q1. Consider the distribution of the daily wages of 50 employees of a factory as given below. Determine the mean of the daily wages of the workers of a factory using the approximate method.

Daily wages (in Rs)	500-520	520-540	540-560	560-580	580-600
Number of workers	12	14	8	6	10

Q.2The given table shows the expenditure on the food of 25 households in a locality. Find the mean of daily expenditure on food using a suitable method.

Daily expenditure (in Rs)	100-150	150-200	200-250	250-300	300-350

Number of households	4	5	12	2	2

Q3 Study the following graph and answer the questions carefully:

Q4The difference of number of students passed to those failed is minimum in which year?

Q5. What is an approximate percentage of students failed during 5 years?

Q6. What is an average number of students failed in the school in last 5 years?

Unit 12: measures of central tendency and dispersion

12.1Introduction

12.2 Unit Objectives

12.3 Understanding Measures of Central Tendency: Mean, Median, Mode

12.4 Measures of Dispersion: Range, Variance, and Standard Deviation

12.5 Applications of Central Tendency and Dispersion in Data Analysis

12.6 Case Studies and Problem Solving in Data Interpretation

12.7 Unit summary

12.8 Check your progress.

12.1 Introduction

In statistics, the central tendency is the descriptive summary of a data set. Through the single value from the dataset, it reflects the centre of the data distribution. Moreover, it does not provide information regarding individual data from the dataset, where it gives a summary of the dataset. Generally, the central tendency of a dataset can be defined using some of the measures in statistics.

Definition

The statistical measure that captures the single value of the entire distribution or dataset is called the central tendency. It seeks to give a precise account of all the data in the distribution.

A statistical tool for summarizing a data set with a single value that represents the middle or center of its distribution is called a measure of central tendency, sometimes referred to as a measure of center or central location. The three primary measures of central tendency are:

- Mean
- Median
- mode

12.2 unit objectives

After studying this unit, you will :

- Define central tendency and requirements of a good measure of central
- tendency
- Define and explain arithmetic mean
- Define median and its calculation
- Define mode and its calculation
- Explain relationship between mean, median and mode

12.3 Understanding the Measures of Central Tendency: Mean, Median, and Mode

Mode
The value that appears the most frequently in a distribution is called the mode.
Examine this dataset, which displays 11 individuals' retirement ages in full years:

54, 54, 54, 55, 56, 57, 57, 58, 58, 60, 60
This table shows a simple frequency distribution of the retirement age data.
Frequency distribution table

Age	Frequency
54	3
55	1
56	1
57	2
58	2
60	2

The most commonly occurring value is 54, therefore the mode of this distribution is 54 years.

Arithmetic mean (A.M)

The arithmetic mean is simply called 'Average'. For the observations $x1$, $x2$, ..., xn

the A.M. is defined as

$$\bar{x} = \text{A.M.} = \frac{\sum_{i=1}^{n} x_i}{n}$$

For simple frequency distribution,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\text{N}}, \quad \text{where N} = \Sigma f_i$$

For the grouped data(frequency distribution), the arithmetic mean is given by

$$\bar{x} = \frac{1}{\text{N}} \Sigma fx,$$

Where f is the frequency, x is the midpoint of the class interval and N the totsl number of observation

Properties of Arithmetic Mean

The arithmetic mean has the following properties:

(i) "Algebraic sum of the deviations of a set of values from their arithmetic mean is zero." If *xi fi* , *i* = 1, 2, ..., *n* is the frequency distribution, then

$$\boxed{\Sigma f(x-\bar{x})=0}$$

(ii) The sum of the squares of the deviations of a set of values is minimum when taken about mean.

$$\boxed{\Sigma f(x-\bar{x})^2 \rightarrow \text{minimum}}$$

(iii) Mean of composite series. If *xi* (*i* = 1, 2, ..., *k*) are the means of *k*-component series of size n_i , (*i* = 1, 2, ..., *k*) respectively, then the mean *x* of the composite series obtained on combining the component series is given by the formula:

$$\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2+\ldots+n_k\bar{x}_k}{n_1+n_2+\ldots+n_k}=\frac{\sum_{i=1}^{k} n_i\bar{x}_i}{\sum_{i=1}^{k} n_i}$$

(iv) The arithmetic mean thus obtained likewise increases or decreases by the same constant value if each value of the variable x is raised or lowered by a constant value.

(v) The arithmetic mean thus obtained is equal to the initial arithmetic mean multiplied or divided by the constant value if the values of the variable are multiplied or divided by a constant value.

Arithmetic Mean's Benefits and Drawbacks

The advantages and disadvantages of the arithmetic mean are as follows:

(a) Arithmetic Mean Advantages

(i) It is most commonly used and widely understood.

(ii) It is simple and easy to calculate.

(iii) It is based on all the observations.

(iv) It is a good measure for comparison. (v) It is adaptable to arithmetic and algebraic treatment

(b) Demerits of Arithmetic Mean

(i) Abnormal and extreme values have a significant impact on the mean value.

(ii) It may not be actually present in the series. For example, the average of 2,

3 and 10 is 5, which is not an observation of the series.

(iii) It can be calculated if certain item is missing. Further, in case of open-end

interval, it is calculated on certain assumption.

(iv) It cannot be located by mere observation.

Calculation of Arithmetic Mean

Mainly three forms of data are available, which are given below:

(i) Individual series or ungrouped data

(ii) Discrete series

(iii) Continuous series

(i) **Calculation of Arithmetic Mean in Individual Series**

The arithmetic mean of a series can be calculated using either the direct method (a) or the shortcut method (also known as the deviation method).

(a) Direct method: The arithmetic mean of a set of n observations x1, x2, ..., xn

is denoted by x and is defined as

$$\bar{x} = \frac{1}{n} \sum x$$

(b) Short-cut method: If the observations and magnitude of the observations

is large, short-cut method is used to reduce the arithmetic calculations.

The formula is

$$\bar{x} = A + \frac{1}{n}\sum d,$$

where A is assumed mean

d is their deviations of the observations from assumed mean (x – A)

and n is the total number of observations***.***

Example 1: Calculate the arithmetic mean for a series of Serum Albumin Levels

(g%) of 15 Pre-school children:

2.90	*3.75*	*3.66*
3.57	*3.30*	*3.76*
3.72	*3.62*	*3.69*
2.98	*3.76*	*3.43*
3.61	*3.38*	*3.76*

Solution: The total of all these values, *i.e.,* $\sum x$= 52.89

Total number of observations (n) = 15

Therefore, the arithmetic mean,

$$\bar{x} = \frac{1}{n}\sum x = \frac{52.89}{15}$$

= 3.53%

***Example 2*:** Calculate the arithmetic mean from the number of the spikelets

per spike in wheat:

Number of spikelets per spike: 18, 20, 21, 19, 28, 22, 29, 30, 31, 35.

Solution: Assumed mean = 24

	Spikelets per spike 'x'	Deviations from the assumed mean $d = x - A$
	18	$18 - 24 = -6$
	20	$20 - 24 = -4$
	21	$21 - 24 = -3$
	19	$19 - 24 = -5$
	28	$28 - 24 = 4$
	22	$22 - 24 = -2$
	29	$29 - 24 = 5$
	30	$30 - 24 = 6$
	31	$31 - 24 = 7$
	35	$35 - 24 = 11$
Total	$\Sigma x = 253$	$\Sigma d = 13$

(a) ***Direct method:*** Here $\sum x = 253$, $n = 10$

$$\bar{x} = \frac{1}{n}\Sigma x = \frac{253}{10} = 25.3$$

(b) ***Short-cut method:*** Here $\sum d = 13$

$$\bar{x} = A + \frac{1}{n}\Sigma d$$

= 24 + 13/10 = 25.3

Example 3: Calculate the arithmetic mean of Haemoglobin values (g%) of 26 normal children

Haemoglobin value (g%)	No. of children
10.4	*1*
11.2	*3*
11.8	*4*
12.9	*7*
13.5	*5*
13.8	*4*
14.2	*2*
Total	*26*

Solution:

Haemoglobin value 'x' (g%)	No. of children 'f'	fx
10.4	1	10.4
11.2	3	33.6
11.8	4	47.2
12.9	7	90.3
13.5	5	67.5
13.8	4	55.2
14.2	2	28.4
Total	$N = 26$	$\Sigma fx = 332.6$

$$\bar{x} = \frac{1}{N}\Sigma fx = \frac{332.6}{26} = 12.79 \text{ (g\%)}$$

Example 4: Calculate the arithmetic mean of protein intake of 400 families

Protein intake/ consumption unit/day (g)	*15–25*	*25–35*	*35–45*	*45–55*	*55–65*	*65–75*	*75–85*
No. of families	*30*	*40*	*100*	*110*	*80*	*30*	*10*

Solution;

Protein intake/ consumption unit/day (g) 'class interval'	**No. of families 'f'**	**Mid-point of the class-interval 'x'**	**Multiply f and x 'fx'**
15–25	30	20	600
25–35	40	30	1200
35–45	100	40	4000
45–55	110	50	5500
55–65	80	60	4800
65–75	30	70	2100
75–85	10	80	800
Total	$N = 400$		$\Sigma fx = 19000$

By direct method, $\bar{x} = \frac{1}{N}\Sigma fx = \frac{19000}{400} = 47.50$ g.

(b) by shortcut method: according to this method,

$$\bar{x} = A + \frac{1}{N}\Sigma fd \;;$$

where A = assumed mean

$d = x - A$, x is the mid-point

N = ∑f

(c) step deviation method

$$\bar{x} = A + \frac{h}{N}\Sigma fd,$$

Where A = assumed mean

d = $\frac{x-A}{h}$

and h = magnitude of the class-interval.

$\frac{1}{2}$(20+25)= 22.5

Calculation of Median

The data must be sorted in ascending order of magnitude in order to calculate the median. The median is the middle number 43 when the total number of observations is odd. The median is determined by taking the average of the two middle values if the number of observations is even.

$$\boxed{\text{Median} = \text{size of} \left(\frac{n+1}{2}\right)^{\text{th}} \text{item}}$$

If the number of observations is even, then median is obtained by taking the arithmetic mean of the two middle terms or

$$\text{Median} = \text{size of} \left[\frac{\left(\frac{n}{2}\right)^{\text{th}} \text{item} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{item}}{2}\right]$$

Example 12: Calculate the median from the data recorded on the number of clusters per plant in a pulse crop:

Number of clusters = 10, 18, 17, 19, 10, 15, 11, 17, 12.

Solution: Arrange the data in ascending order, i.e.,

10, 10, 11, 12, 15, 17, 17, 18, 19, n = 9, i.e., odd

Median = size of ((n+1)/2)th item

= size of ((n+1)/2)th item

= size of 5th item

= 15

Example 14: Calculate the median from the following frequency distribution:

No. of branches	*0–3*	*3–6*	*6–9*	*9–12*	*12–15*
No. of plants	*4*	*8*	*22*	*10*	*4*

Solution:

No. of branches 'classes'	No. of plants frequency 'f'	Cumulative frequency 'c.f.'
0–3	4	4
3–6	8	12
6–9	22	34
9–12	10	44
12–15	4	48
	48	

Here N/2 = 48/2 = 24

c.f. just greater than 24 is 34 and corresponding class is 6–9. Therefore, the median

class is 6–9.

Example 15: Calculate the median from the following frequency distribution:

Class-intervals	0–10	10–30	30–60	60–80	80–90	90–100
Frequency	5	16	30	12	6	1

Solution

Class-intervals	Frequency 'f'	Cumulative frequency c.f.
0–10	5	5
10–30	16	21
30–60	30	51
60–80	12	63
80–90	6	69
90–100	1	70
	$N = 70$	

Here N/2= 70/2 =35

c.f. just greater than 35 is 51 and corresponding class is 30–60. Therefore, the median

class is 30–60.

$l = 30$, $h = 30$, $f = 30$, $C = 21$.

Median = $30 + \frac{30}{30}(35 - 21) = 30 + 14 = 44$

Hence the median is 44

Mode

Calculation of Mode

Mode is calculated by different methods, depending upon the nature of the series. (i) **Calculation of Mode in Individual Observations*:*** For determining the mode, count the number of items, the various values repeat themselves and the value occuring the maximum number of times is the modal value.

Example 17: Calculate the mode of the following data relating to the weights of a sample of 10 experimental animals:

S. No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Weight (kg.)	*10*	*11*	*10*	*12*	*12*	*11*	*9*	*8*	*11*	*11*

Solution

Weight (*kg.*)	No. of animals
8	1
9	1
10	2
11	4
12	1
13	1

Since the item 11 occurs the maximum number of times, *i.e.,* 4, hence the modal value is 11.

12.2 Measures of Dispersion: Range, variance, and Standard deviation

Dispersion in Statistics

Dispersion in statistics is a way to describe how spread out or scattered the data is around an average value. Knowing how close or how far apart the data points are from one another is helpful.

Dispersion shows the variability or consistency in a set of data. There are different measures of dispersion like range, variance, and standard deviation.

Measure of Dispersion in Statistics

Measures of Dispersion measure the scattering of the data. It tells us how the values are distributed in the data set. In statistics, we define the measure of dispersion as various parameters that are used to define the various attributes of the data.

These measures of dispersion capture variation between different values of the data.

Example: Find the range of the data set 10, 20, 15, 0, 100.

Solution:

- Smallest Value in the data = 0
- Largest Value in the data = 100

Thus, the range of the data set is,

R = 100 – 0

R = 100

Range for Ungrouped Data

Finding the data set's smallest and largest values through observation is the first step in determining the range for the ungrouped data set. The difference between them provides the range of ungrouped data.

The following example will help us understand this:

Example: Find out the range for the following observations, 20, 24, 31, 17, 45, 39, 51, 61.

Solution:

- Largest Value = 61
- Smallest Value = 17

Thus, the range of the data set is

Range = 61 – 17 = 44

Measures of Dispersion Formula

Measures of Dispersion Formulas are used to tell us about the various parameters of the data. Various formulas related to the measures of dispersion are discussed in the table below.

Absolute Measures of Dispersion	**Related Formulas**
Formulae of Measures of Dispersion	
Range	**H – S** where, H is the Largest Value and S is the Smallest Value
Variance	Population Variance, **$\sigma2 = \Sigma(xi-\mu)2 /n$** Sample Variance, **$S2 = \Sigma(xi-\mu)2 /(n-1)$** where, μ is the mean and n is the number of observation
Standard Deviation	**S.D. = $\sqrt{(\sigma2)}$**
Mean Deviation	**$\mu = (x – a)/n$** where, **a** is the central value(mean, median, mode) and **n** is the number of observation

Measures of Dispersion and Central Tendency

Both Measures of Dispersion and Measures of Central Tendency are used to describe different characteristics of data. Here's a comparison between them:

Central Tendency	**Measure of Dispersion**
Central Tendency vs. Measure of Dispersion	
Central Tendency is a term used for the	Measure of Distribution is used to

Central Tendency	**Measure of Dispersion**
numbers that quantify the properties of the data set.	quantify the variability of the data of dispersion.
Measure of Central tendency include, 1. Mean ● Median 5. Mode	Various parameters included for the measure of dispersion are, • Range • Variance ● Standard Deviation • Mean Deviation

Examples on Measures of Dispersion

Let's solve some questions on the Measures of Dispersion.

Examples 1: Find out the range for the following observations. {20, 42, 13, 71, 54, 93, 15, 16}

Solution:

Given,

- Largest Value of Observation = 71
- Smallest Value of Observation = 13

Thus, the range of the data set is,

Range = 71 – 13

Range = 58

Example 2: Find out the range for the following frequency distribution table for the marks scored by class 10 students.

Marks Intervals	**Number of Students**

10-20	**8**
20-30	**25**
30-40	**9**

Solution:

Given,

- Largest Value: Take the Higher Limit of the Highest Class = 40
- Smallest Value: Take the Lower Limit of the Lowest Class = 10

Range = 40 – 10

Range = 30

Thus, the range of the data set is 30.

Example 3: Calculate the mean deviation for the given ungrouped data {-5, -4, 0, 4, 5}

Solution:

Mean(μ) = {(-5)+(-4)+(0)+(4)+(5)}/5

μ = 0/5 = 0

M.D = $\frac{\sum|d|}{n}$

=> M.D = $\frac{|(-5-0)+|(-4-0)|+|(0-0)|+|(4-0)|+|(5-0)|}{5}$

=> M.D. = (5+4+0+4+5)/5

=> M.D.= 18/5 = 3.6

Standard deviation

Standard deviation =

$$\boxed{(\sigma) = \sqrt{\frac{1}{n}\Sigma(x-\bar{x})^2}} \quad \text{or} \quad \sqrt{\frac{1}{n}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]}$$

For grouped data(frequency distribution)

Standard deviation

$$\boxed{(\sigma) = \sqrt{\frac{1}{N}\Sigma f(x-\bar{x})^2}} \quad \text{or} \quad \sqrt{\frac{1}{N}\left[\Sigma fx^2 - \frac{(\Sigma fx)^2}{N}\right]}$$

$$\boxed{\text{Coefficient of standard deviation} = \frac{\text{Standard deviation}}{\text{Mean}} = \frac{\sigma}{\bar{x}}}$$

Example 4: The following are the patients per day attended by 10 famous doctors of Lucknow city :

25, 34, 48, 36, 42, 70, 30, 60, 45, 50

Find the standard deviation:

solution

x	$(x-\overline{x})$	$(x-\overline{x})^2$
25	– 19	361
34	– 10	100
48	4	16
36	– 8	64
42	– 2	4
70	26	676
30	– 14	196
60	16	256
45	1	1
50	6	36
$\Sigma x = 440$	$\Sigma(x-\overline{x}) = 0$	$\Sigma(x-\overline{x})^2 = 1710$

$$\overline{x} = \frac{1}{n}\Sigma x = \frac{1}{10} \times 440 = 44$$

$$\sigma = \sqrt{\frac{1}{n}\Sigma(x-\overline{x})^2} = \sqrt{\frac{1}{10}\times 1710} = \sqrt{171}$$

$$\sigma = 13.076$$

Example 5: Find the arithmetic mean and standard deviation of the height of 10 persons given below

:

Height (in cm) : 160, 160, 161, 162, 163, 163, 163, 164, 164, 170.

Solution:

Calculation of Arithmetic mean and standard deviation

Height (cm) 'x'	$d = x - A$, where $A = 162$	$d^2 = (x - A)^2$
160	– 2	4
160	– 2	4
161	– 1	1
162	0	0
163	1	1
163	1	1
163	1	1
164	2	4
164	2	4
170	8	64
	$\Sigma d = 10$	$\Sigma d^2 = 84$

$$\bar{x} = A + \frac{1}{n}\Sigma d = 162 + \frac{10}{10} = 162 + 1 = 163$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2} = \sqrt{\frac{84}{10} - \left(\frac{10}{10}\right)^2} = \sqrt{8.4 - 1} = \sqrt{7.4}$$

Mean, $= 2.72.$

Variance

The square of standard deviation is called variance and is denoted by σ2.

Symbolically:

$$\text{Variance} = \sigma^2$$

$$\sigma = \sqrt{\text{variance}}$$

The formula for calculating variance is given by:

$$\sigma^2 = \frac{1}{n}\Sigma(x-\bar{x})^2 \qquad \text{or} \qquad \sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

(for individual observations)

$$\sigma^2 = \frac{1}{N}\Sigma f(x-\bar{x})^2 \qquad \text{or} \qquad \sigma^2 = \frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2$$

(for discrete and continuous series)

Example 7: Calculate the mean, standard deviation and variance of the following frequency distribution

Height in inches	*95–105*	*105–115*	*115–125*	*125–135*	*135–145*
No. of children	*19*	*23*	*36*	*70*	*52*

Solution : let assumed mean = 130

Height in inches class-interval	**No. of children 'f'**	**Mid-point 'x'**	$d = \frac{x-A}{h}$	fd	fd^2
95–105	19	100	$\frac{100-130}{10} = -3$	– 57	171
105–115	23	110	$\frac{110-130}{10} = -2$	– 46	92
115–125	36	120	$\frac{120-130}{10} = -1$	– 36	36
125–135	70	130	$\frac{130-130}{10} = 0$	0	0
135–145	52	140	$\frac{140-130}{10} = 1$	52	52
	$N = 200$			$\Sigma fd = -87$	$\Sigma fd^2 = 351$

Mean=

$$A + \frac{h}{N}\Sigma fd = 130 + \frac{10}{200} \times (-87) = 130 - 4.35 = 125.65$$

125.65.

Standard deviation

$$(\sigma) = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times h$$

$$= \sqrt{\frac{351}{200} - \left(\frac{-87}{200}\right)^2} \times 10$$

$$= \sqrt{1.75 - 0.189} \times 10$$

$$= \sqrt{1.56} \times 10$$

$$= 1.2489 \times 10$$

$$\sigma = 12.489$$

Variance $\sigma = 12.489$

Variance $\sigma^2 = (12.489^2) = 155.97$

Variance = 156

12.3 application of central tendency and dispersion in data analysis

In data analysis, **central tendency** and **dispersion** are essential concepts that help summarize and interpret data effectively. They provide valuable insights into the nature of the data and allow analysts to make informed decisions. Here’s how each of these concepts is applied in data analysis:

1. Central Tendency

Central tendency measures the center or typical value of a dataset. It gives us a single value that represents the entire distribution of the data.

Measures of Central Tendency:

- **Mean**: The arithmetic average of all values in the dataset.

- **Median**: The middle value when the data is sorted in ascending or descending order. It is less sensitive to extreme values (outliers) than the mean.
- **Mode**: The value that occurs most frequently in the dataset. Useful for categorical or discrete data.

Applications:

- **Summarizing Data**: Central tendency helps in summarizing large datasets by representing a typical value, allowing for easier comparison between groups.
- **Understanding the Distribution**: The mean, median, and mode can reveal whether the data is symmetric or skewed. For example, if the mean is greater than the median, the data may be positively skewed (right tail).
- **Decision Making**: In fields like economics, business, or social sciences, central tendency measures help managers, policymakers, and researchers understand the general trend of a population or sample. For example, understanding the average income of a population or the most frequent outcome in an experiment.
- **Predictive Models**: In machine learning, central tendency can be used for baseline predictions or in feature engineering, where the mean or median might be used to fill missing values.

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2. Dispersion

Dispersion refers to the spread or variability of the data points in a dataset. It tells us how much the data values differ from the central value (mean, median, etc.).

Measures of Dispersion:

- Range: The difference between the highest and lowest values in the dataset.
- Variance: The average squared deviation from the mean. It gives an overall measure of the spread of the data.

- Standard Deviation: The square root of the variance. It is in the same unit as the data and provides a more interpretable measure of spread.

- Interquartile Range (IQR): The difference between the first and third quartiles (Q1 and Q3), capturing the spread of the middle 50% of the data.

Applications:

- **Understanding Data Spread**: Dispersion shows the degree of variability in the data. For example, in finance, knowing the standard deviation of stock returns is crucial to understanding the risk (volatility) of an investment.
- **Data Quality**: High dispersion can indicate a data set with significant variability or inconsistency. It can highlight outliers or extreme values that need further investigation or correction.
- **Identifying Patterns**: Low dispersion (low standard deviation) indicates that the data points are consistently close to the central tendency, while high dispersion suggests a more diverse set of values, which could indicate more complex patterns or variability in the underlying process.

- Outlier Detection: A large range or high standard deviation might signal outliers or unusual variations in the data. In this case, dispersion helps in identifying data points that deviate significantly from the rest of the dataset.

Applications in Data Analysis:

1. **Descriptive Statistics**: Both central tendency and dispersion are foundational for descriptive statistics. They provide a quick summary of data, helping analysts identify general trends, variation, and outliers.
2. **Comparing Datasets**: By comparing measures of central tendency and dispersion across different datasets, analysts can evaluate which dataset has more consistency or variation. For instance, comparing the mean and standard deviation of exam scores across different schools can indicate which school's performance is more consistent.

3. **Data Cleaning**: By analyzing the range and standard deviation, data analysts can detect outliers or errors that may need to be addressed to improve the dataset's accuracy.

4. **Statistical Inference**: In hypothesis testing, central tendency and dispersion are used to make inferences about the population from a sample. For example, the standard error of the mean is derived from the standard deviation and sample size, allowing analysts to quantify uncertainty in their estimates.

5. **Predictive Analytics**: Both concepts are fundamental when developing predictive models. Measures like variance help to assess model accuracy and reliability, while central tendency helps establish baseline predictions.

Example:

Suppose you're analyzing the salaries of employees in a company:

- **Central Tendency**: The mean salary might be $60,000, which provides a typical salary estimate. The median salary might be $58,000, indicating that half of the employees earn more and half earn less. The mode might be $50,000 if it's the most common salary.

- **Dispersion**: The standard deviation might be $10,000, indicating that most employees earn between $50,000 and $70,000, but some earn much higher or lower. A large variance might show significant disparities in salaries between employees.

By understanding both central tendency and dispersion, you can make informed decisions about the company's pay structure, identify outliers (e.g., extremely high or low salaries), and determine how typical or diverse employee salaries are.

In summary, central tendency provides insight into the typical value of a dataset, while dispersion offers a deeper understanding of how spread out or variable the data is. Both are critical for interpreting and making informed decisions from data.

12.4 practice questions

Question 1: Given the dataset: [12, 15, 18, 22, 26], calculate the mean.

Question 2: Find the median of the dataset: [5, 2, 9, 4, 7].

Question 3: Identify the mode in the dataset: [3, 7, 3, 9, 12, 7, 3].

Calculate the mean for the following dataset: [-5, 10, -3, 8, 12].

Determine the median of the dataset: [23, 18, 12, 35, 27, 19].

Question 7: A teacher recorded the following test scores for her class: [85, 78, 92, 88, 84, 90, 79, 81, 87, 93, 78, 84].

- Calculate the mean score.
- Determine the median score.
- Identify the mode(s) if any.

Check your progress

Problem 1: Calculating the Mean

1. Find the mean of the following data set: [4, 8, 15, 16, 23, 42].
2. A class of students scored the following marks in a test: [55, 67, 89, 45, 90, 75, 88]. What is the mean score?

Problem 2: Finding the Median

1. Determine the median of the data set: [10, 2, 38, 23, 38, 23, 21].
2. Calculate the median for the following scores: [85, 89, 93, 67, 88, 91, 92, 78].

Problem 3: Identifying the Mode

1. Identify the mode in the following data set: [7, 7, 8, 10, 12, 8, 7].
2. What is the mode for this set of data: [45, 56, 45, 67, 56, 78, 89, 45]?

Unit 13: Introduction to Probability

13.1 Introduction

The mathematical concept of probability deals with events and uses numerical values between 0 and 1 to quantify how likely they are to occur. A higher probability means that the event is more likely to occur. Essentially, it is a ratio of the specified event to the total number of events.

Concepts of Probability are used in various real life scenarios :

- **Stock Market:** Investors and analysts often study these parameters and use probabilistic models to understand trends and patterns for the movement of stock price.
- **Insurance:** Insurance companies use probability models to estimate the likelihood of various events to manage this risk, and set premiums accordingly.
- **Weather Forecasting:** Meteorologists use probability to predict the likelihood of various weather events, such as rain, snow, storms, or temperature changes.

13.2 Unit objectives:

After studying this lesson you will learn

- Understand the meaning of random experiment

- determine P(E) if P(E) is given;
- state that for the probability P(E), $0 \leq P(E) \leq 1$;
- Apply the concept of probability in solving problems based on tossing a coin throwing a die, drawing a card from a well shuffled deck of playing cards, etc.

13.3: concept of an experiment, random experiment and simple space.

In science and engineering if we conduct an experiment and repeat it under identical conditions, we get almost the same result every time. Such an experiments are known as deterministic experiments.

But there are experiments which, when repeated under identical conditions, do not produce the same outcome every time. For example, if we toss a fair coin, we may get a head or a tail. Now, if we make further trials, i.e. toss the coin again and again, the outcome of each trial depends on the chance, and it is not the same each time. Sometimes the head appears and sometimes the tail appears . these experiments are known as Random experiments.

Experimental probability, or empirical probability, refers to estimating the likelihood of an event happening based on the results of actual experiments or observations. It is calculated by dividing the number of favorable outcomes by the total number of trials or experiments conducted.

To understand this better, imagine flipping a coin. The theoretical probability of landing heads is 50% or 1/2. However, if you actually flip the coin 100 times and record the outcomes, you might get heads 48 times. The experimental probability of getting heads would then be 48/100 or 0.48.

Formula for Experimental Probability

The experimental Probability for Event A can be calculated as follows:

P(E) = (Number of times an event occur in an experiment) / (Total number of Trials)

Examples of Experimental Probability

The probability that a head would occur when tossing a coin ten times and recording four head and six tail would be as follows:

P(H) = 4/10

Similarly, the Probability of Occurrence of Tails on tossing a coin:

P(T) = 6/10

13.3.2 Concept of Random experiment and its outcomes.

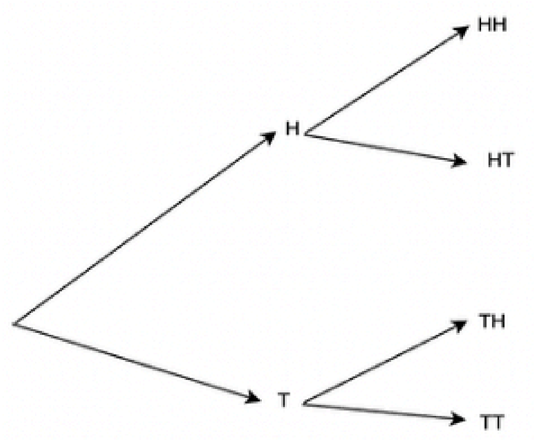
Random Experiments

The underlying assumption and a key aspect of probability theory is that it experiments must be random. It is assumed that the experiment can be performed infinitely many times under the same conditions. This assumption becomes important because both classical probability theory and frequency-based probability theory rely on this assumption as they look for long-term behaviours in the experiments.

A random experiment is an experiment in which the outcome cannot be determined with certainty in advance. It is impossible to predict the result when a die is rolled. This is an example of a random experiment. Each random experiment must fulfil the following two conditions:

- Experiment can be repeated a number of times under the same conditions.
- Outcome of the experiment cannot be predicted beforehand with certainty.

For example: A coin is tossed two times and the outcomes are recorded. The possible outcomes are:
HH, HT, TH, TT



Notice that there are four possible outcomes in this experiment and none of them can be predicted beforehand. Thus, in the context of our experiment, the sample space is a universal set. It is denoted as S. For the above experiment, S = {HT, HH, TH, TT}

Solved examples on Random Experiment:

Q1.Consider an experiment of rolling a die and then tossing a coin. Draw the sample space.

Solution:

A die when rolled given {1,2,3,4,5,6} as result while tossing a coin gives {H,T}.

Sample space S = {(1, H); (2, H); (3, H); (4,H); (5,H); (6,H); (1, T); (2, T); (3, T); (4, T); (5, T); (6, T)}

Q2. To determine the probability of getting 2 heads in three tosses of a coin.

Solution:

A coin toss gives either Head (H) or Tails (T).

Outcome of three coin tosses will be triplets of Heads and Tails. So, the sample space is,

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Counting the number of favorable outcomes, there are three outcomes with two heads.

Probability = $\frac{favourable\ number\ of\ outcomes}{total\ number\ of\ outcomes}$

Therefore P = 3/8

Q3. A fair coin is tossed three times. What is the probability of getting exactly two heads?

Solution:

Step 1: Identify the sample space.

The sample space consists of 8 possible outcomes: HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

Step 2: Count favorable outcomes.

There are 3 outcomes with exactly two heads: HHT, HTH, THH

Step 3: Calculate the probability.

Probability is calculated by dividing the total number of outcomes by the number of favorable ones.

= 3 / 8

= 0.375 or 37.5%

- (1, 3), (1, 4), . . . (6, 6)}.

  Q2. How many possible outcomes are there in the sample space when flipping a coin three times?

  Solution:

  There are 23 = 8 elements in the sample space when flipping a coin three times.

  Q3. How many different pairs of marbles can you draw from a jar containing 20 red marbles and 30 blue marbles, when two marbles are drawn without replacement?
  Solution:

  There are $20C_1$ (choosing 1 red marble) ×30C1 (choosing 1 blue marble) = 20 × 30 = 600 different pairs you can get when drawing two marbles without replacement.

  Q4. How many different PIN combinations are possible for a 4-digit PIN code, where each digit can range from 0 to 9?

  Solution:

There are 10,000 possible PIN combinations for a 4-digit PIN code when each digit can be 0-9.

13.4 Independent and mutually Exclusive events

Examples of Mutually Exclusive Events

Coin toss:

- **Event A:** Getting Heads
- **Event B:** Getting Tails

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These are mutually exclusive because a coin can't be both heads and tails in a single toss.

Rolling a Die:

- **Event A:** Rolling an even number (2, 4, 6)
- **Event B:** Rolling an odd number (1, 3, 5)

These are mutually exclusive because a number can't be both even and odd.

Selecting a Card from a Deck:

- **Event A:** Drawing a Heart
- **Event B:** Drawing a Spade

These are mutually exclusive because a single card can't be both a heart and a spade.

Examples of Independent events

Coin Tosses:

- **Event A:** Getting heads on the first toss
- **Event B:** Getting tails on the second toss

These are independent because the outcome of the first toss doesn't affect the second.

Drawing Cards with Replacement: -

Event A: Selecting a heart on the first draw.

Event B: Selecting a spade on the second draw (with the first card replaced).

These are independent because replacing the card keeps the probabilities constant.

Weather in Different Cities:

- **Event A:** Rain in New York
- **Event B:** Sunshine in Los Angeles

13.5 Unit summary:

A random experiment is one that has multiple possible outcomes, and its result cannot be predicted with certainty before it is conducted. An event consists of one or more outcomes, while an elementary event refers to a single outcome. The probability of an event is the ratio of favorable outcomes to the total number of equally likely possible outcomes. The probability of an event ranges from 0 to 1, where (P(E) = 0) indicates an impossible event, and (P(E) = 1) indicates a sure event. The sum of probabilities of all elementary events in an experiment equals 1, and for complementary events (E) and (E'), the sum of their probabilities is always 1.

13.6 Check your progress

1. Determine the likelihood of receiving the number 5 in a single die roll.
2. One die toss is made. What is the likelihood that it displays? (i) a seven? (ii) a figure below five?
3. A card is randomly selected from a 52-card deck. How likely is it that this card will be a king?

4. A number between 0 and 20 is selected. How likely is it that this selected integer is a prime number?

Unit 14: Introduction to linear programming

14.1 Introduction:

The term "linear programming" is composed of two words: "linear" refers to the relationship between different variables of degree one in a problem, while "programming" indicates the step-by-step procedure used to solve these types of problems.

The basic components of a linear programming (LP) problem are:

- **Decision Variables:** Variables you want to determine to achieve the optimal solution.
- **Objective Function: M**athematical equation that represents the goal you want to achieve
- **Constraints:** Limitations or restrictions that your decision variables must follow.
- **Non-Negativity Restrictions:** In some real-world scenarios, decision variables cannot be negative

14.1.2 Problems on LPP

Linear Programming Problems (LPP) focuses on optimizing a linear function to determine the best possible solution. The optimal solution can either be the maximum or the minimum value.

In LPP, the linear functions are called objective functions. An objective function can have multiple variables, which are subjected to conditions and have to satisfy the linear constraints.

Linear Programming Formula

A linear programming problem typically includes the following components:

- Decision Variables: These are the variables (e.g., x and y) that determine the output of the linear programming problem and represent the final solution.
- Objective Function: Represented by Z, this is the linear function that needs to be either maximized or minimized, depending on the problem's goal. The objective function is optimized based on the given conditions.
- Constraints: These are the restrictions placed on the decision variables, which limit their possible values. Constraints define the feasible region within which the solution must lie.

- Non-Negative Restrictions: The decision variables are typically subject to non-negativity restrictions, meaning that they must be greater than or equal to zero .

Objective Function: Z = ax + by

Constraints: cx + dy ≥ e, px + qy ≤ r

Non-Negative restrictions: x ≥ 0, y ≥ 0

Solving Linear Programming Problems?

The steps involved are as follows:

1. Define the Decision Variables: List and indicate the problem's decision variables.

2. Create the Objective Function: Formulate the objective function and determine if it needs to be maximized or minimized.

3. List the Constraints: Write down all the constraints associated with the problem.

4. Apply Non-Negativity Constraints: Ensure that the decision variables are subject to non-negative restrictions.

5. Solve the Problem: Use an appropriate method, typically either the simplex or graphical method, to solve the linear programming problem.

Linear Programming Methods

There are various methods used to solve linear programming problems, with the two most commonly used being:

1. Simplex Method
2. Graphical Method

14.2 Unit objectives

Upon completing the study of this unit, you should be able to:

- describe the principle of simplex method
- discuss the simplex computation
- explain two phase and M-method of computation
- Analyze the relationship between variables

14.3 Principle of solving equations by Simplex Methods

One of the most popular methods for resolving linear programming issues is the simplex method. This approach entails going through a number of steps repeatedly until the best solution is discovered. The following are the steps involved in applying the simplex method to a linear programming problem:

1. Problem Definition: Formulate the linear programming problem based on the given constraints and objective function.
2. Convert Inequalities into Equalities: Convert all inequality constraints into equalities by adding slack variables where needed.
3. Create the Initial Simplex Table: Represent each constraint as an equation in a row and include the objective function in the last row, forming the simplex table.
4. Identify the Pivot Column: Find the most negative value in the bottom row. The column associated with this value is the pivot column.
5. Determine the Pivot Row: Divide the values in the rightmost column by the corresponding values in the pivot column (excluding the objective row). The row with the smallest ratio becomes the pivot row. The pivot element is at the intersection of the pivot row and pivot column.
6. Update the Pivot Column: Use matrix operations and the pivot element to make all other entries in the pivot column equal to zero.
7. Check for Optimality: If all entries in the bottom row are non-negative, the solution is optimal. Otherwise, repeat the process starting from Step 4.
8. Obtain the Final Solution: The simplex table derived in the final step provides the solution to the problem.

14.4 Solutions to Minimization of Problems

Example 1: A finished product must weigh exactly 150 grams. The two raw materials used in manufacturing the product A, with a cost of Rs2 per unit and B with a cost of Rs 8 per unit. At least 14 units of B and not more than 20 units of A must be used. Each unit of A and B weighs 5 and 10 grams respectively. To reduce costs, how much of each kind of raw material should be used for each unit of the finished product? Apply the Simplex method.

Solution: the given problem can be expressed as LPP as

Minimize $Z= 2X_1 + 8X_2$

Subject to

$5X_1 +10X_2 =150$

$X_1 \leq 20$

$X_2 \geq 14$

$X_1, X_2 \geq 0$

Substituting the values $X_2 = 14 + X_3$ and introducing the necessary slack and artificial variables, we have

Minimize $Z= 2x_1+8x_3+112 +MA1+0S1$

$5x_1 +10x_3+ A1 = 10$

$X_1 + S_1 = 20$

$X_1, x_2, x_3, S_1, A_1 \geq 0$

The solution is contained in the table:

Basis		x_1	x_3	A_1	S_1	bi	bi/ a_{ij}
A1	M	5	10	1	0	10	1
S1	0	1	0	0	1	20	∞

Cj	2	8	M	0
Solution	0	0	10	20
Δj	2 – 5 M	8 – 10M		

Basis		X_1	X_3	A_1	S_1	bi	bi/a_{ij}
X3	8	1/2	1	1/10	0	1	2
X4	0	1	0	0	1	20	20
Cj		2	8	M	0		
Solution		0	1	0	20		
		-2	0	$M - \frac{8}{10}$	0		

Basis		X_1	X_3	A_1	S_1	bi
X_1	2	1	2	1/5	0	2
X_4	0	0	-2	-1/5	1	18
Cj		2	8	M	0	
Solution		2	0	0	18	
Δj		0	4	M– 2/5	0	

Thus, the optimal solution is x1 = 2 units, x2 = 14+ 0 = 14 units, total cost = 2* 2+8*14 = Rs. 116.

14.5 Solution to Maximization of problems

Consider a company that operates three machines to produce three different products. Each unit of Product A takes 3 hours on Machine I, 2 hours on Machine II, and 1 hour on Machine III. Product B requires 4 hours on Machine I, 1 hour on Machine II, and 3 hours on Machine III for each unit. Product C, on the other hand, needs 2 hours on each of the three machines. The contribution margins per unit for Products A, B, and C are ₹30, ₹40, and ₹35, respectively. The total available machine hours are 90 for Machine I, 54 for Machine II, and 93 for Machine III.

(1) Formulate the above problem as a linear programming problem.

(2) Obtain optimal solution to the problem by using the simplex method. Which of the three products shall not by produced by the firm? Why?

(3) Calculate the percentage of capacity utilization in the optimal solution.

(4) What are the shadow prices of the machines hours?

(5) Is the optimal solution degenerate?

(1)

Let x1, x2 and x3 represent the output of products A,B and C respectively. With the given contributions margins, resource requirements and availability, the LPP can be represented as follows:

Maximise $Z = 30X_1 + 40\,X_2 + 32\,X_3$ Contributions

Subject to

$3\,x_1 + 4x_2 + 2x_3 \leq 90$ machine I

$2\,x_1 + x_2 + 2x_3 \leq 54$

$X_1 + 3x_2 + 2x_3 \leq 93$

$X_1, x_2, x_3 \geq 0$

(ii) solving the question using simplex methods, we first introduce the slack variables S_1, S_2, S_3 and express the problem in standard form as follows:

Maximize $Z = 30\,x_1 + 40\,x_2 + 35\,x_3 + 0S_1 + 0S_3$

Subject to $3x_1 + 4x_2 + 2x_3 + S2 = 90$ machine I

$2x_1 + x_2 + 2x_3 + S2 = 54$

$X_1 + 3x_2 + 2x_3 + S3 = 93$

$X_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

The solution is given in tables

Basis		x1	x2	x3	S1	S2	S3		bi	bi/aij
S1	0	3	4	2	1	0	0		90	45/2
S2	0	2	1	2	0	1	0		54	54
S3	0	1	3	2	0	0	1		93	31
Cj		30	40	35	0	0	0			
Solution		0	0	0	90	54	93			
Δj		30	40	35	0	0	0	Z = 0		

Basis		x1	x2	x3	S1	S2	S3	bi	bi/aij
X2	40	3/4	1	1/2	¼	0	0	45/2	45
S2	0	5/4	0	3/2	-1/4	1	1	63/2	21
S3	0	-5/4	0	½	-3/4	0	1	51/2	51
Cj		30	40	35	0	0	0		
Solution		0	45/2	0	0	63/2	51/2		
Δj		0	0	15	-10	0	0	Z= 900	

Basis		x1	x2	x3	S1	S2	S3	bi
X2	40	1/3	1	0	1/3	-1/3	0	12
X3	35	5/6	0	1	-1/6	2/3	0	21
S3	0	-5/3	0	0	-2/3	-1/3	1	15
Cj		30	40	35	0	0	0	
Solution		0	12	21	0	0	0	
Δj		-25/2	0	0	-15/2	-10	0	Z= 1215

(iii) The capacity available and the capacity used as per optimal solution is given below

Machine	Capacity	Capacity Unused	Capacity Utilised	Percentage Utilisation
I	90	0	90	100%
II	54	0	54	100%
III	93	15	78	83.78%

(iv) shadow prices of the machine hours are given by the Δj values (ignoring signs) of slack variables,

Accordingly , they are:

Machine I ` Rs 15/2 or Rs 7.50 per hour

Machine II Rs 10 per hour

Machine III Nil

(v) The optimal solution is not degenerate because none of the basic variables has a solution value equal to zero.

14.6 Unit Summary

Linear Programming (LP) is a mathematical technique used to find the optimal solution (such as maximizing profit or minimizing cost) in a mathematical model where the constraints and objectives are expressed through linear equations or inequalities.

Key Components:

1. **Objective Function**: A linear function to be maximized or minimized, e.g., $\mathbf{Z = c_1x_1+c_2x_2+\cdots+c_n x_n}$
2. **Constraints**: A set of linear inequalities or equations that restrict the values of decision variables.
3. **Decision Variables**: Variables representing quantities to be determined.

The Simplex Method

- **Purpose**: A systematic procedure to find the optimal solution to a linear programming problem.
- **Applicability**: Used when there are multiple variables and constraints, and graphical methods are not feasible.

Steps in the Simplex Method

1. **Formulate the Problem**:
   - Define the objective function.
   - Write the constraints as linear inequalities or equations.
   - Identify the non-negativity conditions for all variables ($x_1 \geq 0$).
2. **Convert to Standard Form**:
   - Rewrite the problem to ensure all constraints are equalities by adding slack, surplus, or artificial variables.
   - Ensure all variables have non-negative values.
3. **Set Up the Initial Simplex Tableau**:
   - Construct a tabular representation including coefficients of the objective function, constraints, and slack/surplus variables.
4. **Iterative Optimization**:

Step 1: Identify the Entering Variable:

- Choose the variable with the most negative coefficient in the objective function row (for maximization problems).

Step 2: Identify the Leaving Variable:

- Compute the ratios of the right-hand side to the coefficients of the entering variable in each constraint row.
- The smallest positive ratio determines the leaving variable.

Step 3: Perform Pivoting:

- Update the tableau by making the pivot element (intersection of the entering and leaving variables) equal to 1, and all other elements in its column zero.

5. **Check for Optimality**:
   - Repeat the iterative steps until all coefficients in the objective function row are non-negative (for maximization problems).
   - For minimization, the process is similar, focusing on the least positive coefficients.
6. **Interpret the Results**:
   - Read the values of decision variables and the optimal value of the objective function from the final tableau.

Advantages of the Simplex Method

- Handles large-scale problems efficiently.
- Provides both the optimal solution and sensitivity analysis (shadow prices).

Limitations

- Computationally intensive for extremely large problems.
- Assumes linearity and certainty in relationships.

Applications

- Resource allocation in manufacturing.
- Transportation and logistics.
- Financial portfolio optimization.
- Scheduling and workforce management.

14.7 check your Progress

1. Consider the linear programming problem

$$\begin{aligned} &\text{Maximise } 5x_1 + 4x_2 \\ &\text{Subject to :} \\ &\qquad x_1 \leqslant 7 \\ &\qquad x_1 - x_2 \leqslant 8 \\ &\qquad x_1 \geqslant 0, \; x_2 \geqslant 0. \end{aligned}$$

2. Solve the following Linear programming by simplex method.

$$\begin{aligned} &\text{Minimise } 200x_1 + 300x_2 \\ &\text{Subject to :} \\ &\qquad 2x_1 + 3x_2 \geqslant 1200 \\ &\qquad x_1 + x_2 \leqslant 400 \\ &\qquad 2x_1 + 3/2x_2 \geqslant 900 \\ &\qquad x_1 \geqslant 0, \; x_2 \geqslant 0. \end{aligned}$$

3. Solve the following problem by simplex method and give your comments.

Maximize $3x_1 + 2x_2$

Subject to:

$x_1 - x_2 \leq 1$

$3X_1 - 2x_2 \leq 6$

$X_1 \geq 0, \; x2 \geq 0$

Unit 15: Inventory Management

15.1 Introduction

Inventories are vital for maintaining the seamless functioning of manufacturing and retail organizations. They usually include raw materials, work-in-progress items, spare parts or consumables, and finished goods. Although it is not necessary for an organization to maintain every category of inventory, efficiently managing the inventory they hold is crucial, as a substantial portion of the organization's financial resources is often tied up in it. Various departments within an organization often have differing views on inventory management due to their unique responsibilities. For instance, the sales department may focus on keeping ample stock to fulfill customer demands quickly. Similarly, the production department may prioritize maintaining adequate material stocks to ensure uninterrupted operations. On the other hand, the finance department may push for minimizing inventory investment to free up funds for other, potentially more lucrative, opportunities.

15.2 Unit Objectives

By the end of this unit, learners will be able to:

1. Understand the Importance of Inventory in identifying the different roles inventory plays in operational efficiency.
2. Explore EOQ Models
3. Grasp the Concept of Safety Stock
4. Summarize Key Concept of inventory types, EOQ models, and safety stock.
5. Apply the knowledge gained to real-world inventory management situations.
6. Evaluate Progress through self-check exercises and problem-solving activities.

15.3 Types of inventory

Inventories are maintained for various purposes in organizations. Broadly, they can be classified into five types:

1. **Movement Inventories (Pipeline Inventories)**

   These inventories arise because of the time needed to transport goods from one place to another. For instance, coal being transported from a mining area to an industrial town remains in transit and is unavailable for immediate use, such as power generation or industrial burning, until it arrives at its destination.

2. **Buffer Inventories**

   - Buffer inventories serve as a safeguard against uncertainties in demand and supply.
   - They account for the unpredictability of demand and lead times, ensuring that shortages are minimized and customer goodwill is preserved.
   - The optimal level of buffer stock can be determined using specific methodologies to balance cost and availability.

3. **Anticipation Inventories**

- These inventories are held to prepare for predictable future demand.
- Examples include stocking up on crackers before Diwali, umbrellas before the rainy season, or increasing inventory when a strike is anticipated.
- The goal is to maintain steady production over time, avoiding periods of excessive overtime or inactivity due to fluctuating demand.
- These inventories compensate for varying work rates of machines or personnel, acting as buffers to prevent one stage of production from halting due to delays in another.
- For instance, if a machine breaks down, decoupling inventories allow other machines to continue operating without interruption.

4. **Cycle Inventories**

    - These inventories arise because purchasing and production are typically done in batches rather than continuously.
    - Cycle inventories ensure that materials are available in adequate quantities between replenishment cycles, optimizing production and reducing the frequency of orders.

15.4 Types of EOQ models:

(i) The traditional model of EOQ:
This model is based on the following basic assumptions:
(a) The demand for the item is certain, constant and continuous over time.

(b) The lead time, which is the period between placing an order and receiving the delivery, is considered constant. If the lead time is zero, the item is delivered immediately..
(c) Within the range of the quantities to be ordered, the per unit holding cost and the ordering cost (per order) are constant and are independent of the quantity ordered.
(d) the purchase price of the item is constant, that is to say, no discounts is available on purchases of large lots.
(e) the inventory is replenished immediately as the stock level reaches exactly equal to

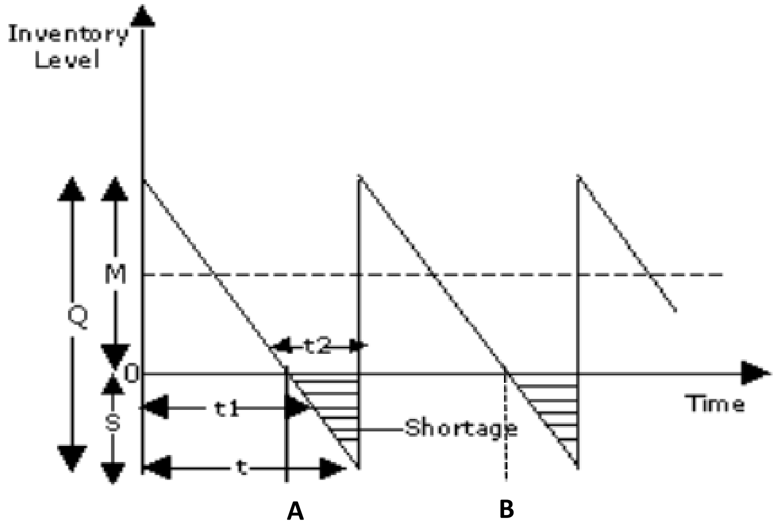


Fig. Inventory profile of classical EOQ model.

Assume that we start with the Q stock at time 0. This will be used up at a daily rate of d units. A new order could be placed and the inventory could be acquired at point t1 if the stock could be refilled instantly, that is, if the lead time was zero. Likewise, at point t2, an order would be placed once this stock was used up. In contrast, if the lead time is positive, we would place a new order at point A, meaning that we would have consumed the stock by the time the new delivery was scheduled to arrive, provided we had an amount in stock equal to the demand during the lead time. This is called the reorder level.

I. The inventory cycle is the period of time between two consecutive order placement points (for instance, A and B) or the amount of time needed to use up the entire lot of received items (Q). t is represented in the figure by t. The maximum inventory held would be Q while the minimum be zero, and hence the average inventory level would be equal to Q/2.
For determining the optimum order quantity, we have to consider two types of cost, i.e. the ordering and the holding cost. Since the purchase price of the item is uniform, it does not affect the decision as to the quantity of the item to be ordered for purchase and, hence, is irrelevant for the purpose.

According to this, assuming a period of 1 year, be

T(Q) = O(Q) + H(Q)

Where, Q = the ordering quantity

T(Q) = total annual inventor cost

O(Q) = total annual ordering cost.

H(Q) = total annual holding cost.

Example. A manufacturing company has determined from an analysis of its accounting and production data for a certain part that (i) its demands is 9000 units per annum and is uniformly distributed over the year, (ii) its cost price is Rs2 per unit, (iii) its ordering cost is Rs. 40 per order, (iv) the inventory carrying charge is 9% of the inventory value.

Further, it is known that the lead time is uniform and equals to 8 working days, and the total working days in a year is 300.

Determine

(a) The economic order quantity, EOQ;

(b) The optimal numbers of orders per annum;

(c) The total ordering and holding cost associated with the policy of ordering an amount equal to EOQ;

(d) The re-order level;

(e) The number of days stock at the re-order level;

(f) The length of the inventory cycle.

(g) The amount of savings that would be possible by switching to the policy of ordering EOQ determined in (a) from the present policy of ordering the requirements of this part thrice a year; and

(h) The increase in the total cost associated with ordering (i) 20% more, and (ii) 40% less than the EOQ.

Solution:

We are given that D = 9,000 units/year, A = Rs40/order, I = 0.09, c = Rs2/unit, and therefore, $h = i \times c = 0.9 \times 2 = 0.18$, also lead time = 8 working days, and total working days in the year = 300

(a) EOQ, $Q = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \times 40 \times 9{,}000}{0.18}} = 2{,}000$ units

(b) optimum number of orders per year,

$N = \frac{D}{Q}$ = 9,000/2,000 = 4.5

(c) Total variable cost = T(Q) = $\sqrt{(2ADh)}$ = $\sqrt{2 \times 40 \times 9{,}000 \times 18}$

= Rs 360

(d) Re-order level = lead time in days $\times$ demand per day

= 8 $\times \frac{9{,}0000}{30}$ = 240 units.

(e) No of days stock at the re-order level = 8(equal to lead time)

(f) length of the inventory cycle, T = $\frac{Q}{D} = \frac{2000}{9000}$ = 0.222 $\times$ 300

= 66.7 days.

(g) For the present policy of an order quantity = 3,000 units

Ordering cost = 40 $\times$ 3= Rs120

Holding cost = (3000/2) $\times$ 0.18 = Rs. 270

T (3,000) = 120+270 = Rs 390

Therefore saving in cost = Rs390 – Rs360 = Rs30 per year.

(h) (i) Ordering 20% higher than EOQ:

Ordering Quantity = $\frac{120}{100} \times 2{,}000$ = 2,400 umits

With Q = 2,000 and Q = 2,400, k = 2,4000/2,000 = 1.2

We have, $\frac{T(Q)}{T(Q*)} = \frac{1}{2}\left(\frac{1}{k} + \text{k}\right) = \frac{1}{2}\left(\frac{1}{1.2} + 1.2\right)$ = 61/60

Thus, the cost would increase by 1/60th or 360 $\times \frac{1}{60}$ = Rs6.

(ii) Ordering 40% lower than EOQ:

In such a situation, k = 0.60, and

$\frac{T(Q)}{T(Q*)} = \frac{1}{2}\left(\frac{1}{0.60} + 0.60\right) = 17/(15)$.

Thus the increase in cost would be 2/15th over the cost for EOQ, and would equal 360 * 2/15 = Rs48.

(ii) EOQ with Price Breaks

The total cost function T(Q) incorporating price breaks, is given by:

$$T(Q) = \frac{D}{Q} A + \frac{Q}{2} h + c_i D$$

Where:

- Q: Order quantity
- D: Annual demand
- A: Ordering cost of the item/order
- h: Holding cost per unit/year
- Ci: Unit cost, which changes based on the quantity ordered

Price Break Intervals

- For $Q < q_1$, the unit price is C_0
- For $q_1 \le Q \le q_2$, the unit price is C_2
- For $q_2 \le Q \le q_3$, the unit price is C_3, and so on.
- For $Q \ge q_n$, the unit price is C_{n-1}.

Therefore, the price per unit is C_0 when the order quantity (Q) is less than the specific size q1, and C1 when the order quantity is at least equal to q1 but less than q_2. and so on. Let's look at the example that follows:

A TV tube where D = 2,000 units per annum, A = Rs150 per order, and h = Rs2.40 per unit annum. Suppose now that the supplier informs that if the order size is at least 800 units, he is prepared to supply the tubes at a discounted price of Rs9.80 per tube. We shall now examine whether the offer of the supplier is attractive or not.

With the EOQ = 500 units already established, the total cost involved would be,

$$T(Q) = \frac{2000}{5000} \times 150 + 500/2 \times 2.40 + 10 \times 2{,}000$$

= Rs21,200

The Price of the tubes has been taken to be Rs10 per tube as the lower price is available only for the order sizes of at least 800 units, thus, we shall determine the total cost involved at the minimum quantity at which the discount is available. At Q = 800, we have,

$$T(Q) = \frac{2000}{800} \times 150 + \frac{800}{2} \times 2.40 + 9.80 \times 2{,}000$$

= Rs20,935

Q. A company offered the following price breaks for the order quantity as follows:

1. Order quantity < 500, price Rs10.00

2. Order quantity > 500, price Rs9.00

The cost per order is ₹100.00, and the holding cost is 25% of the unit price. Calculate the economic order quantity (EOQ) given that the annual demand is 2,500 units.

Solution:

Annual demand D = 2500 units per year

Ordering cost C_0 = Rs.100 per order

Inventory holding rate I = 0.25 of unit price

Case1:

Let EOQ is less 500 for the inventory holding cost.

$$C_h = \frac{25}{100} \times 10 = 2.5$$

For this case, EOQ = $\sqrt{\frac{2DC0}{Ch}} = \sqrt{\frac{2\times2500\times100}{2.5}} = 447.2$

Total inventory cost (TIC) = $\sqrt{2DC0Ch} = \sqrt{2 \times 2500 \times 0.25 \times 10 \times 2500}$

= 1118.03

Case2:

Let EOQ is greater than 500, for this case

C_h = 25/100 $\times 9 = 2.25$

For this case, EOQ = $\sqrt{\frac{2DC0}{Ch}} = \sqrt{\frac{2\times2500\times100}{2.25}}$ = 471.4

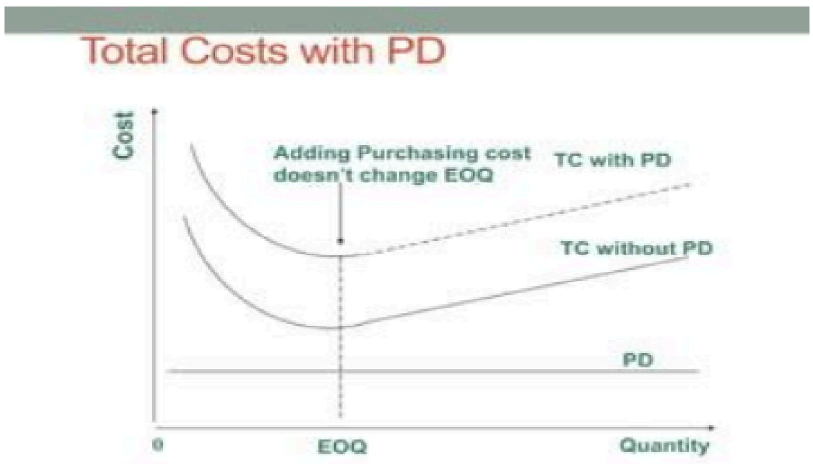
This is against the assumption made, so correct EOQ =447.2 = 447 units

When price break occurs, TIC at Q = 500

TIC = (Average inventory $\times$ unit inventory cost) + (No. of orders per year $\times$ cost of the order).

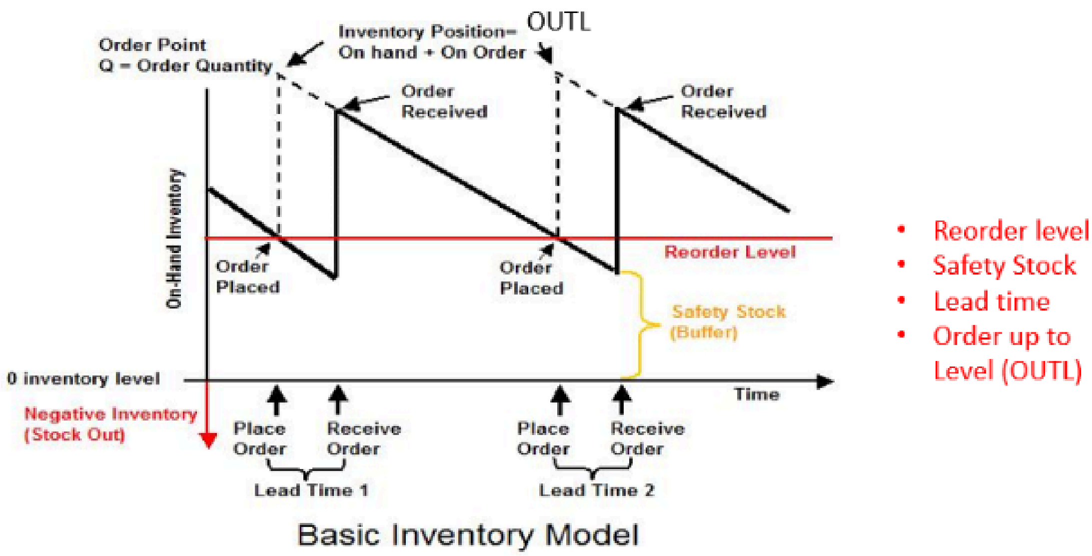
=> $\frac{Q}{2} \times ch + \frac{D}{Q} \times C0$ = 500/2 $\times 9 \times 0.25 + \frac{2500}{500} \times 100$ = 1062.5

Hence the quantity to be ordered is 500.



15.5 Concept of Safety stock with examples.

Deterministic models assume that inventory is replenished as soon as it is depleted, so under ideal conditions, there is no need for additional stock because supplies would arrive exactly when the inventory reaches zero, avoiding stock outs (unless purposefully planned).



The safety stock is an important constituent of the re-order point. In the fixed orer quantity inventory system under consideration, the re-order level would be determined as the expected demand of the item during lead time*plus the safety stock.

For instance, you are provided with the following details about a product.

Annual usage = 20,000 units

Ordering cost = Rs 160 per order

Carrying cost = 20% of the average inventory investment

Unit cost = Rs 2

Lead time = 10 working days

Total working days = 250 per annum.

Given this level, what kind of safety stock would you suggest? Also determine (a) the re-order level when the safety stock level suggested by you is kept in stock, (b) average level of the inventory stock held, and (c) the ordering and carrying costs associated with this fixed order inventory policy.

We know, safety stock, SS max DDLT – average DDLT

With maximum demand per day = 150 units

Average demand per day = 20,000/250 = 80 units

Lead time = 10 days

We have, Maximum DDLT = 150 * 10 = 1,500 units

Average DDLT = 80*100 = 800 units

SS = 1500 – 800 = 700 units

(a) Re-order level = SS + average DDLT= 700 + 800 = 1500 units

(b) Average stock level = SS + Q/2

From the given information, Q= $\sqrt{\frac{2\times160\times20{,}000}{0.20\times2}}$ = 4,000 units

Thus, average stock level = 700 + 4,000/2 = 2,700 units

(c) ordering cost = 20,000/4,000 *160 = Rs800

Carrying cost =2,700 * 0.40 = Rs1,080

Therefore total cost = 800+10800 = Rs1,880

15.6 unit summary

This unit provided a comprehensive understanding of inventory management, focusing on its critical role in optimizing business operations. It introduced the various types of inventory—raw materials, work-in-progress, finished goods, and MRO supplies—highlighting their significance Throughout various stages of production and supply chain management, we have examined the two categories of costs linked to inventory management, namely: (a) Ordering cost

(b) Carrying costs.

The Economic Order Quantity (EOQ) refers to the quantity of inventory that minimizes the combined costs of ordering and carrying inventory. The inventory level at which a firm places an order for replenishment is called the reorder point, which depends on:

(a) Lead time

(b) the usage rate.

15.7 Check your progress

Q1. A manufacturer purchases specific equipment from external suppliers at ₹30 per unit, with an annual requirement of 800 units. The following details are also provided:

Annual return on investment: 10%

Rent, insurance, and taxes per unit per year: ₹1

Cost of placing an order: ₹100

Calculate the Economic Order Quantity (EOQ).. (**Ans.200 units**)

Q2. What is your understanding of Maximum Level, Minimum Level, and Reordering Level? Using the provided data, calculate these levels.

Re-order quantity	**1,500 units**
Re-order period	4 to 6 weeks
Maximum consumption	400 units per week
Normal consumption	300 units per week
Minimum consumption	250 units per week.

Ans. (**Reorder quantity 2,400 units: Maximum level 2,900 units: Minimum level 900 units**)